National Council on Teacher Quality

Technical Manual for Elementary Mathematics

Supporting documentation for the NCTQ 2022 *Teacher Prep Review: Preparation for Teaching Elementary Mathematics* report and NCTQ Elementary Mathematics standard

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Introduction

Teacher preparation programs must provide elementary teacher candidates with the opportunities to gain the mathematics content and pedagogical knowledge necessary for effective instruction. The National Council on Teacher Quality (NCTQ) recognizes that institutions can follow many paths to provide these opportunities, but ultimately, programs must ensure that candidates attain this essential knowledge.

Programs establish requirements for all enrolled candidates seeking an initial elementary[1] teaching license, which typically include prescribed and elective coursework as well as supervised practice. The requirements also can include taking and passing an elementary licensure test prior to admission to the program or completing prerequisite courses before scheduling program-specific courses.[2] Successful completion of program requirements should serve as evidence that candidates have demonstrated at least minimal mastery of a body of knowledge and skills.

By evaluating the programs that produce more than 95% of traditionally-prepared elementary teachers,[3] NCTQ cannot directly measure the mastery of the content and pedagogical knowledge candidates obtain (this is a function typically reserved for state-adopted licensure assessments) or the application to teaching (a function typically reserved for teacher evaluation). NCTQ can review the ***breadth of the requirements*** established by programs and ***instructional time allocated for essential topics***, and measure these against widely accepted understandings of relevant and important constructs. (See Appendix A for a discussion of the research rationale for the NCTQ Elementary Mathematics standard.)

[1] Definitions of, and terminology for, "elementary education" vary from state to state and impact licensure requirements. Many states define elementary education as K-6 (or preK-6) but others use different grade bands.
[2] Most states also require candidates to pass prescribed licensure test(s) for initial licensure.
[3] Based on a three-year average of "Prepared by Area" figures in Title II releases from 2019-2021. (https://title2.ed.gov/Public/Home.aspx)

Process to Revise the Elementary Mathematics Standard

Since its initial publication in 2013, the *Teacher Prep Review* (TPR) has grown and evolved, while steadfastly adhering to the goal of providing external, high-quality feedback to teacher preparation programs. It has now been 17 years since the initial groundwork leading to the *Review*; the time had come to refresh the research underpinning our standards, revisit our methodologies, and examine how the *Review* can have the most impact. In 2021, NCTQ launched the effort to refresh our existing Elementary Mathematics standard with deep engagement from the field.

The standards revision process included multiple points of external engagement:

- Gathering input on the content of the standard from an **Expert Advisory Panel** of mathematics experts, leaders of teacher prep programs, and practitioners in districts and schools.
- Inviting **public comment** on the draft changes to the standard through a survey shared with teacher preparation program leaders and mathematics faculty, school district leaders, state leaders, advocates, and others.
- Gathering input on the analysis and scoring process from a **Technical Advisory Group** of mathematics experts, economists, and psychometricians.

The purpose of the Elementary Mathematics standard revision process was threefold:

1. Facilitate a transparent process that is more inclusive of feedback from external stakeholders than was the case for the original standards.
2. Clarify what we can claim about a program's preparation of teachers based on available evidence.
3. Ensure feedback is clear and actionable for programs.

This revision sought to keep pace with changes in the field and to rectify concerns over the utility of the previous standard (e.g., the use of textbooks and the complexity of the scoring system).

Defining Critical Content and Pedagogical Knowledge

NCTQ revised the Elementary Mathematics standard to reflect current expectations for elementary students (based on widely adopted elementary mathematics standards) and the knowledge and skills needed by teachers (teacher mathematics standards discussed below). In examining teacher preparation programs, NCTQ gathered program requirements as evidence to support the following claim.

> *In order that elementary teachers are able to deliver equitable and effective mathematics instruction, teacher preparation programs provide their elementary candidates with both the content and pedagogical knowledge specified in commonly accepted mathematics education standards.*

NCTQ drew on multiple sources to describe the mathematics content programs should address in required coursework. Expectations for elementary students were drawn from the Common Core State Standards for Mathematics (CCSS-M) and the National Council of Teachers of Mathematics (NCTM) Principles and Standards for School Mathematics. Teacher standards and recommendations for teacher preparation were drawn from the Conference Board of the Mathematical Sciences' Mathematical Education for Teachers II (MET II), the American Statistical Association's Guidelines and Assessments for Instruction in Education

(GAISE), and the Council for the Accreditation of Educator Preparation (CAEP) K-6 Elementary Teacher Preparation Standards. A synthesis of these recommendations was reviewed by NCTQ's Expert Advisory Panel (EAP) and was shared with the field for feedback through an Open Comment Period.

- The EAP agreed that programs should address both specialized mathematics content knowledge and mathematics pedagogy. This was confirmed by almost all of the 275 educators who completed the Open Comment Survey- 96% agreed or strongly agreed that programs should teach specialized mathematics content knowledge and mathematics pedagogy.
- The EAP discussed and agreed that the NCTQ review should focus on four key mathematics content topics-Numbers & Operations, Algebraic Thinking, Geometry & Measurement, and Data Analysis & Probability. From the Open Comment Survey, 93% of respondents agreed or strongly agreed that the four topics are the "right areas of specialized content knowledge" on which to evaluate elementary programs.

NCTQ's revised approach considered how programs dedicate instructional time across required coursework to each of four key mathematics topics (examples provided to illustrate the scope of the topics):

1. Numbers & Operations
    - Understand numbers, ways of representing numbers, relationships among numbers, and number systems.
    - Understand meanings of operations and how they relate to one another.
    - Compute fluently and make reasonable estimates.

2. Algebraic Thinking
    - Understand patterns, relations, and functions
    - Represent and analyze mathematical situations and structures using algebraic symbols.
    - Use mathematical models to represent and understand quantitative relationships.
    - Analyze change in various contexts.

3. Geometry & Measurement
    - Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
    - Specify locations and describe spatial relationships using coordinate geometry and other representational systems.
    - Apply transformations and use symmetry to analyze mathematical situations.
    - Use visualization, spatial reasoning, and geometric modeling to solve problems.
    - Understand measurable attributes of objects and the units, systems, and processes of measurement.
    - Apply appropriate techniques, tools, and formulas to determine measurements.

4. Data Analysis & Probability
    - Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them.
    - Select and use appropriate statistical methods to analyze data.
    - Develop and evaluate inferences and predictions that are based on data.
    - Understand and apply basic concepts of probability.

Additionally, to ensure that candidates not only learn the content but also how to teach each of these four topics well, the Elementary Mathematics standard considered the instructional time dedicated to a fifth topic:

5. **Mathematics Pedagogy**

NCTQ gathered mathematics course requirements and admissions requirements for elementary teacher candidates and evaluated if courses or tests for admission into the program[4] address *mathematics content knowledge* and/or *pedagogical knowledge for teaching mathematics*.

Sample of Teacher Preparation Programs

For the 2022 Elementary Mathematics *Teacher Prep Review*, the sample included 1,111 programs in 49 states[5] and the District of Columbia — both undergraduate and graduate — housed in 941 institutions of higher education (IHEs) that offered elementary teacher preparation that led to initial state licensure or certification.

Approximately half of the programs (577 or 52% of the 1,111 programs) are housed in public IHEs. NCTQ reviewed 811 undergraduate and 300 graduate programs.[6] Non-traditional programs are not included in this report, but maybe evaluated in a separate report to be released at a later date.

The sample generally includes all public institutions that actively produce elementary teachers and all private institutions with an annual production of at least 10 elementary teachers. NCTQ examined both undergraduate and graduate elementary programs at nearly 18% of the institutions (170 of 941 IHEs). Many of the remaining institutions offered only an undergraduate or a graduate elementary program.

While not a census of all programs in the nation or a representative sampling,[7] the 1,111 programs are a diverse subset representing institutions that produce over 95% of traditionally-prepared elementary teachers[8] in the nation and illustrate the variety of programs preparing elementary teacher candidates.

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[4] "Admissions testing" refers to testing requirements to be admitted to the teacher preparation program and not the college or university. More specifically, it refers to the program requiring candidates to take and pass an elementary licensure test with a separate mathematics subtest as a condition for admission to the program.

[5] Alaska was not included because, at the time of selecting the sample (2019) for this cycle of the *Teacher Prep Review*, the University of Alaska system was scheduled to be consolidated into a single institution, for which there was not a clear set of requirements. Future releases will be inclusive of Alaska programs.

[6] Eighteen programs (two undergraduate and 16 graduate) in the sample did not require any mathematics content or pedagogy courses based on initial review. Based on follow-up with programs, one graduate program provided information indicating mathematics coursework was required.

[7] The 2020 Title 2 Report includes 1,882 IHE-based providers in academic year 2018-19. Not all providers necessarily offer elementary teacher preparation programs. (https://title2.ed.gov/Public/Home.aspx)

[8] Based on a three-year average of "Prepared by Area" figures in Title II releases from 2019-2021 (https://title2.ed.gov/Public/Home.aspx)

Gathering Evidence

NCTQ's analysis focused on programs' responsibility to ensure that elementary teacher candidates possess knowledge of the key mathematics topics and the ability to teach those topics to future students. Programs can construct the curricula to require courses that cover the key topics or require candidates to demonstrate mastery of the key topics for entry into the program. To better reflect the program requirements for elementary teacher candidates, NCTQ followed a multi-step process that allowed programs input into the evaluation process. The initial step in the review process was to determine:

- If the program requires candidates to pass an elementary content knowledge licensure test that includes a separate mathematics subtest as a condition of admissions.
- The complete set of course requirements that address elementary mathematics content and pedagogy.

To be considered "required," the course must exist as a universal requirement for all candidates enrolled in the program. If admissions into the program required specific prerequisites (i.e., MATH 101, and not "three credits of college-level mathematics"), the course(s) were considered "required" for the program. If the program required an admissions test[9] (i.e., passing an elementary licensure test) and the test included a separately scored mathematics subtest (rather than a test or subtest that combined mathematics with other subjects), the program was (a) acknowledged for ensuring candidates have demonstrated mastery of key mathematics content topics and (b) credited with partial instructional hours for the content topics.

A team of analysts used course catalogs and degree plans to determine the required coursework for each elementary program in the sample. A separate team of expert mathematics analysts, who undergo a rigorous vetting and training process, read course titles and descriptions to pinpoint all teacher audience courses, inclusive of those that focus on both content and pedagogy, which are flagged for analysis. General audience mathematics courses (College Algebra, for example) are excluded from analysis.

The next step was to gather evidence regarding the content estimated to be covered by each course. With over 1,100 sampled programs and multiple required courses for most programs, NCTQ could not observe each course or interview each professor. Course descriptions were gathered as evidence of the content of the course, and syllabi were used whenever available.

- The EAP understood the limitations of the data sources—course syllabi and course descriptions—available to NCTQ but also understood that more in-depth sources (e.g., course observations, lesson materials) were not available and/or not practical for a review of 1,100+ programs. From the Open Comment Survey, 75% of respondents agreed or strongly agreed that "course syllabi accurately reflect the majority of content taught in the course" and 60% of respondents agreed or strongly agreed that "analyzing course descriptions is acceptable" if course syllabi are not available.

Across programs, NCTQ analysts reviewed syllabi or course descriptions for 2,958 courses. Course syllabi were available for 834 unique courses from 373 programs; course descriptions were available for all

[9] Twenty-three programs require an elementary licensure test with a separate mathematics subtest to be passed before or soon after admissions to the program.

courses. Of the 2,911 unique course descriptions,[10] only 74 (from 47 programs) were too vague to allow for coding of estimated instructional hours).

Nearly all of the 1,111 programs in the sample required one or more mathematics content or pedagogy courses for elementary teacher candidates. Table 1 summarizes the number of required courses for undergraduate and graduate programs.

Table 1. Number of Required Courses, by type of program

	Number of programs	0	1	2	3	4	5+
Undergraduate	811	2 (<1%)	69 (9%)	148 (19%)	319 (40%)	181 (21%)	92 (11%)
Graduate	300	15 (5%)	190 (63%)	62 (21%)	22 (7%)	7 (2%)	4 (1%)

Nearly three-quarters of the undergraduate programs (592 or 73%) require three or more mathematics courses. However, the overwhelming majority of graduate programs (267 or 89%) require two or fewer courses.

Coding Instructional Hours

NCTQ expert analysts coded syllabi and course descriptions to determine the elementary mathematics topics that are addressed in each course. This approach relied on the assumption that the number of references to a topic reflects the proportion of course time dedicated to that topic. Steps to validate this assumption are described in the section on "Syllabus-Description Comparability" below.

Analysts coded courses following the structure described in Figure 1 to determine the distribution of instructional hours for each required course. (Examples of the coding process are provided in Appendix B.) When evaluating course descriptions, two analysts code each description independently, bringing in a third analyst to adjudicate any significant differences. When evaluating course syllabi (which typically contain more information), a random sample was coded by two analysts for real-time monitoring of interrater reliability.

[10] A small number of the courses (47 courses) were required for both the undergraduate and graduate programs at the institution.

Figure 1. Basic Coding Structure

<table>
<tr><th colspan="2">When coding a **Course Description**</th><th colspan="2">When coding a **Syllabus**</th></tr>
<tr><td rowspan="3">Reference Count</td><td rowspan="3">Count of the total number of references to each identifiable topic (e.g., Numbers & Operations)</td><td>Unit Type</td><td>Identify the type of calendar (daily, weekly, etc.) used in the syllabus</td></tr>
<tr><td>Unit Count</td><td>Identify the total number of units (days, weeks, etc.) that are defined in the course schedule</td></tr>
<tr><td>Reference Count</td><td>For **each unit**, count of the total number of references to each identifiable topic</td></tr>
<tr><th colspan="2">Within the course description or details presented in the syllabus…</th><th colspan="2">Within **each unit specified in the syllabus**…</th></tr>
<tr><td>1. Numbers & Operations</td><td colspan="3">Count of the number of references to Numbers & Operations including any subtopics within that domain</td></tr>
<tr><td>2. Algebraic Thinking</td><td colspan="3">Count of the number of references to Algebraic Thinking, including any subtopics with that domain</td></tr>
<tr><td>3. Geometry & Measurement</td><td colspan="3">Count of the number of references to Geometry & Measurement including any subtopics with that domain</td></tr>
<tr><td>4. Data Analysis & Probability</td><td colspan="3">Count of the number of references to Data Analysis & Probability, including any subtopics with that domain</td></tr>
<tr><td>5. Mathematics Pedagogy</td><td colspan="3">Count of the number of references to mathematics instructional approaches</td></tr>
<tr><td>6. Other Content Topics[11]</td><td colspan="3">Count of the number of references that address mathematics content topics outside any of the four content areas (e.g., references to trigonometry or calculus)</td></tr>
<tr><td>7. Other Pedagogy Topics</td><td colspan="3">Count the number of references that address general pedagogy or subject-specific pedagogy for subjects other than mathematics.</td></tr>
</table>

Interrater Agreement

Prior to providing programs with initial findings, two analysts independently coded a subset of 107 course syllabi. These syllabi represented a random selection of 16% of all syllabi in NCTQ's possession at the time.

Rater agreement for course syllabi was examined two ways (see Table 2). The first was a binary identification of the four mathematics topics and mathematical pedagogy (the syllabi did or did not reference the topic). Agreement between analysts that the syllabus referenced a topic was above 90% for all topics. The second way was to look at the correlations between the analysts' estimated instructional time dedicated to a topic. The correlations were above 0.96 for all topics.

Course descriptions for all courses were separately coded by two analysts. Again, rater agreement was examined two ways for the 2,765[12] unique courses where the description offered sufficient details (see Table 2). The first was binary identification of the four mathematics content topics and mathematical

[11] While coded, "Other Content Topics" and "Other Pedagogy Topics" were not used to evaluate the Elementary Mathematics Standard.
[12] Course count prior to providing programs with initial analysis under the standard. Some TPPs identified additional courses that were included in final analysis and reporting.

pedagogy. Agreement between the pair of analysts that the description referenced a topic was 97% or higher for all topics. The second was the correlations between the analysts' estimated instructional time dedicated to a topic. The correlations were at or above 0.97 for all topics.

Table 2. Interrater Agreement

	Syllabus Coding (107 courses)		Description Coding (2,765 courses)	
	Binary Agreement	Instructional Hours Correlation	Binary Agreement	Instructional Hours Correlation
N&O	94.4%	0.99	98.7%	0.99
AT	91.6%	0.96	97.7%	0.97
G&M	94.4%	0.99	99.5%	0.99
DA&P	95.3%	0.97	99.4%	0.99
MP	90.7%	0.96	97.5%	0.99

NOTE: N&O = Numbers & Operations, AT = Algebraic Thinking, G&M = Geometry & Measurement, DA&P = Data Analysis & Probability, and MP = Mathematics Pedagogy

Syllabus-Description Comparability

NCTQ attempted to collect syllabi for elementary mathematics courses through a variety of means. Programs that did not initially submit syllabi in response to NCTQ's query also had the opportunity to review their grades based on course descriptions and submit additional course materials. However, for roughly 70% of the courses, NCTQ was not able to obtain a course syllabus and analysis was based upon the course description.

Syllabi, by design, include more detailed information regarding the topics being covered and the amount of instructional time (typically number of course meetings) focused on each topic. Course descriptions are high-level summaries of the topics covered by the course. The assumption supporting the description-coding process is that the text of the description can be parsed into references to mathematics topics and the percentage of references to each topic estimates the percentage of instructional time. The estimated instructional time from a course description is a proxy for the more exact estimate that could be calculated if a syllabus was available.

To determine if using course descriptions was an acceptable proxy, NCTQ analyzed both syllabi and course descriptions from 656 courses (from 322 programs) where both syllabi and course descriptions were available to determine comparability. The first level of analysis was to determine if the same topics could be identified as being covered/not covered using syllabus and description coding. This level of analysis offers binary coding; if a topic was referenced once, it was classified as covered in the course. The second level of analysis is to compare the estimated instructional time calculated from the syllabus and description coding. The correlation between the estimated instructional time from syllabus and description coding was a measure of the strength of the relationship between what can be estimated from the two data sources.

The agreement rate for identifying any coverage of a topic in a course between course descriptions and syllabi ranged from 70% for Algebraic Thinking to 88% for Geometry & Measurement (see Table 3). The correlations between the estimated instructional hours ranged from 0.64 for Algebraic Thinking to 0.88 for Geometry & Measurement. Steps to mitigate the low correlation for Algebraic Thinking are discussed in the *"Scoring Protocol"* section below.

Table 3. Syllabus-Description Comparability (656 Courses)

	Binary Agreement	Instructional Hours Correlation
N&O	86.4%	0.85
AT	70.4%	0.64
G&M	88.1%	0.88
DA&P	84.6%	0.75
MP	80.5%	0.64

NOTE: N&O = Numbers & Operations, AT = Algebraic Thinking, G&M = Geometry & Measurement, DA&P = Data Analysis & Probability, and MP = Mathematics Pedagogy

Instructional Hour Targets

Key to NCTQ's evaluation of programs is the instructional time dedicated to critical content. Meeting the standard is defined by whether programs meet thresholds for the minimum number of instructional hours for each topic. These targets guide the feedback NCTQ can provide colleges and universities, state policymakers, and school districts hiring new teachers. Establishing such thresholds required gathering evidence from multiple sources,[13] including:

- NCTQ's 2008 *No Common Denominator* and the Conference Board of the Mathematical Sciences' 2010 *MET II* reports
- Judgments of NCTQ's Expert Advisory Panels for Mathematics
- A survey of teacher educators and other stakeholders

No Common Denominator and MET II Reports. A NCTQ Mathematics Advisory Group advising on the 2008 *No Common Denominator* report[14] recommended that programs require the equivalent of about two-and-a-half courses (115 instructional hours) to address mathematics content and one course covering mathematics pedagogy. The CBMS's *The Mathematical Education of Teachers II* report[15] recommends 12 semester-hours, or four typical courses, in mathematics for elementary teacher candidates. The recommendation included mathematics content as well as mathematics pedagogy (p. 31). In dividing the 12 semester-hours, the report assigns half the hours to numbers & operations and Algebraic thinking with

[13] See https://www.nctq.org/pages/teacher-prep-review-math-revision-2022 for a more thorough discussion of the evidence gathered to establish instructional hour targets.
[14] Greenberg, J., & Walsh, K. (2008). *No Common Denominator: The Preparation of Elementary Teachers in Mathematics by America's Education Schools*. National Council on Teacher Quality.
[15] Beckmann, S., Chazan, D., Cuoco, A., Fennell, F., & Findell, B. (2012). The Mathematical Education of Teachers II. In *Issues in mathematics education/CBMS, Conference Board of the Mathematical Sciences* (Vol. 17, pp. 1-86).

the remaining half to additional Algebra ideas, geometry, measurement, data analysis and probability. Mathematics pedagogy is not separated from mathematics content when assigning instructional hours.

Input from Experts. In 2021, NCTQ convened an Expert Advisory Panel (EAP) of 10 mathematics education experts to define the construct to be evaluated (discussed above) and to recommend targets for instructional hours. The process for recommending targets was for panelists to first recommend targets for instructional hours independently (Judgment #1) and then to revise their recommendations after discussion (Judgment #2). Six panelists provided both judgments, two panelists only provided Judgment #1, and one panelist provided only Judgment #2. (One panelist did not provide any judgments.) The mean recommended targets from Judgment #2 is presented below; Judgment #1 recommendations were used when Judgment #2 recommendations were not available.

- Numbers & Operations — 39 instructional hours
- Algebra Thinking — 24 instructional hours
- Geometry & Measurement — 27 instructional hours
- Data Analysis & Probability — 17 instructional hours
- Mathematics Pedagogy — 49 instructional hours

Survey of stakeholders. In NCTQ's Open Comment Survey for the revised Elementary Mathematics standard, NCTQ also asked teacher educators, policymakers, and other stakeholders the amount of instructional time that programs should dedicate to each of the mathematics content topics. (For the open comment period, respondents were not asked to provide detailed instructional hours.) The most popular response for 275 respondents is presented below.

- Numbers & Operations — A full course (42% of respondents)
- Algebra Thinking — Roughly half a course (38% of respondents)
- Geometry & Measurement — Roughly half a course (40% of respondents)
- Data Analysis & Probability — Evenly split between roughly half a course and less than half a course (37% of respondents for each option)

Recommended Instructional Hour Targets. The sum of the instructional hours targets is 150 hours, less than previous recommendations in the *No Common Denominator* (160 hours) and the *MET II* (180 hours) reports.[16] The number of instructional hours for Algebraic Thinking is less than the target used for Algebra in the previous version of the standard as is the number of hours for Geometry & Measurement.

Figure 2. Recommended Instructional Hour Targets

	Key Mathematics Topics					
	Numbers & Operations	Algebraic Thinking	Geometry & Measurement	Data Analysis & Probability	Math Pedagogy	Total
Instructional hours	45 hours	20 hours	25 hours	15 hours	45 hours	150 hours
Course equivalent	One full 3-credit course	Just under half of a 3-credit course	Just over half of a 3-credit course	One-third of a 3-credit course	One full 3-credit course	10 credits

[16] Semester credit hours (SCH) were standardized across programs to be 15 instructional hours to adjust for any difference in IHE's academic calendars.

Table 4 summarizes the number of programs meeting or exceeding the instructional hours targets for the four mathematics topics and mathematical pedagogy. Following the initial analysis of the estimated instructional hours, NCTQ contacted the programs with results and invited programs to (a) provide course syllabi for identified courses where only course descriptions were available and (b) identify additional required courses, if they were missed in the original analysis. The results presented in Table 4 represent final reported information.

Table 4. Number (and Percentage) of Programs Meeting or Exceeding Targets[17]

	Undergraduate (771 programs)		Graduate (293 programs)	
	Number of Programs	Percentage	Number of Programs	Percentage
N&O	367	47.6%	19	6.5%
AT	107	13.9%	2	0.7%
G&M	342	44.4%	14	4.8%
DA&P	326	42.3%	11	3.8%
MP	539	69.9%	155	52.9%

NOTE: N&O = Numbers & Operations, AT = Algebraic Thinking, G&M = Geometry & Measurement, DA&P = Data Analysis & Probability, and MP = Mathematics Pedagogy

Summarizing Evidence to Evaluate and Guide Programs

Information on topic coverage across required courses was collated to assign a grade to a program. In addition, if a program required an elementary licensure test with a separate mathematics subtest for admission into the program (or shortly after), the program was credited with instructional hours equivalent to 80% of the recommended target.[18] If a program requires an admission test and additional coursework, instructional hours from both sources are summed together.

The grade is an overall indicator of whether the program provides elementary teacher candidates with both the content and pedagogical knowledge specified in commonly accepted mathematics education standards. In addition to a program's grade, the estimated number of instructional hours for Numbers & Operations, Algebraic Thinking, Geometry & Measurement, Data Analysis & Probability, and Mathematics Pedagogy are provided as evidence of strengths and weaknesses in curricular design. (See above for a description of the instructional hour targets for the key topics.)

[17] Topic-level results are from final, adjudicated program-level grades (discussed below) and after additional information provided by programs. The counts do not include the 47 programs where instructional hours could not be estimated from the available course descriptions.
[18] Instructional hours assigned for requiring an elementary licensure test at admission to the program were 36 hours for Numbers & Operations, 16 hours for Algebraic Thinking, 20 hours for Geometry & Measurement, and 12 hours for Data Analysis & Probability.

Scoring Protocols

For each required mathematics course (content and pedagogy) for elementary teacher candidates, analysts estimated instructional hours focused on each of the topic areas discussed above. Instructional hours were summed across courses to estimate the total instructional hours devoted to each topic. In instances where a course syllabus was available, the distribution of instructional hours for the course was based upon that analysis.

For 23 of the 1,111 programs, candidates were required to take and pass an elementary licensure test that included a separate mathematics subtest for admission into the program (or shortly after). Each of these 23 programs required one or more mathematics courses – 13 of the 23 programs required a course or courses that addressed both mathematics content and pedagogy and the remaining programs required a single course that addressed mathematics pedagogy. The instructional hours assigned for the required test were combined with the hours estimated for the required courses to (a) determine the instructional hours for each of the key mathematics topics and then (b) calculate the program grade.

The grading protocol for programs includes two rules. First, for a program to earn the highest grade, it must require at least 90% of the target hours for each topic (combining Numbers & Operations and Algebraic Thinking). Second, for all other grades, a program's grade is based on the percentage of the overall target instructional hours (see Figure 3).

Figure 3. Program Grading Protocol

Grade	Percentage of Instructional Hour Targets
A+	At least 150 instructional hours across the five topics and 100% of the recommended target hours under *each* of the five topics.
A	At least 135 instructional hours across the five topics and at least 90% of the recommended target hours under *each* of the five topics.
B	At least 120 instructional hours (80%) across the five topics.
C	At least 105 instructional hours (70%) across the five topics.
D	At least 90 instructional hours (60%) across the five topics.
F	Fewer than 90 instructional hours (less than 60%) across the five topics.

To mitigate the risk that course time on Numbers and Operations could not be distinguished from time on Algebraic Thinking, these two topic areas were combined for the purposes of the scoring protocol. Therefore, the instructional hours estimated for Numbers & Operations and Algebraic Thinking were combined[19] and a program was credited for instruction hours up to the combined targets (65 hours).

To protect against the protocol allowing for compensatory results (where instructional hours beyond the target in one topic could compensate for hours below the target in another topic), for grading purposes, hours beyond each topic target were not counted. A program was credited for instructional hours for a topic up to the ***Recommended Instructional Hour Target*** (see Figure 2).

[19] Student standards (e.g., CCSS-M) combine "Operations" and "Algebraic Thinking" through the mid-elementary grades, with Algebra becoming more of a separate topic in later grades. Descriptions of preparation coursework may also combine "Numbers & Operations" and "Algebraic Thinking," making it difficult to separate when determining overall grades for teacher preparation programs.

Adjudication of Program Grades

Under the standard, each program receives both a "course description grade" and a "master grade." In instances where analysis is entirely based upon course descriptions, those two grades are identical. When at least one syllabus is used in analysis, the master grade reflects those findings in place of the course description coding. In these cases, differences may arise between the two grades.

For each program, course description grades were calculated separately based upon the independent coding completed by two analysts. If the grades agreed, no adjudication was required and the topic-level results for one of the analysts were randomly selected for reporting. If the analysts disagreed, the discrepancy was adjudicated to determine the course description grade for the program. For the 1,026 programs for which course description analysis could be completed,[20] the course description grades agreed for 955, or 93.1%, of the programs.

Subsequently, for the 311 programs with at least one course syllabus, the resulting master grade was compared against both the established course description grade and the other analyst's master grade, but only in instances where both analysts evaluated at least one syllabus. Because syllabi were randomly assigned, there were only 40 programs for which both analysts evaluated the same number of syllabi. There was master grade agreement between the analysts for 36 programs (90%).

In cases where the course description grade and master grade were not in agreement, the higher grade was assigned as the final program grade.

For the 23 programs that required an elementary licensure test that included a separate mathematics subtest for admission into the program (or shortly after), the instructional hours for the required courses were combined with the instructional hours assigned for the test. The grades were then adjudicated as described above.

The initial program grades were shared with the programs, and programs were encouraged to supply additional information–identify missing required courses or provide course syllabi–that would provide a more accurate picture of mathematics requirements. Of the 943 institutions that house the 1,111 programs in the sample, 174 (18%) provided additional information. The additional information was reviewed by the NCTQ analysts using the processes described above and the grade for the program was recalculated, if applicable.

Table 5 summarizes the assigned grades for the 1,111 elementary teacher preparation programs examined by NCTQ for the 2022 Elementary Mathematics *Teacher Prep Review*.

[20] Prior to adjudication, at least one course for 85 programs was noted as "couldn't be determined" (CBD) due to a course description that could not be coded. Because the overall instructional hours for these programs was incomplete, they were removed from this comparison.

Table 5. Number (and Percentage) of Programs at Each Grade

	Undergraduate (811 programs)		Graduate (300 programs)	
	Number of Programs	Percentage	Number of Programs	Percentage
A+/A	116	14.3%	5	1.7%
B	292	36.0%	21	7.0%
C	99	12.2%	7	2.3%
D	101	12.5%	12	4.0%
F	162	20.0%	248	82.7%
CBD	41	5.1%	7	2.3%

NOTE: The distribution of grades reflects results first published by NCTQ on May 17, 2022. Programs may submit additional information after the release of the *Teacher Prep Review: Preparation for Teaching Elementary Mathematics* report that changes these results.

Appendix A: Research Rationale for Elementary Mathematics Standard

Does mathematical content knowledge matter for elementary teachers?

In general, elementary students achieve more in math when taught by teachers with greater mathematics content knowledge.[21]

Unfortunately, completing a bachelor's degree or a teacher preparation program does not guarantee that teachers know the math they'll be expected to teach. In fact, one study found that teacher candidates in one state perform poorly on the same topics that students in that state struggled with, although this study could not draw a causal link between teachers' knowledge and that of students.[22]

Another found that many elementary teacher candidates had misconceptions about statistics and probability as they were about to enter student teaching (the culminating experience of most teacher preparation programs).[23] A national survey found that few elementary teachers felt very well-prepared to teach specific elementary mathematics topics, and the proportion who felt very well-prepared declined between 2012 and 2018.[24] This sense of inadequate preparation has persisted for decades; in 2002, detailed surveys of elementary teachers in 60 school districts in Michigan and Ohio[25] indicate that elementary teachers do not feel well prepared to teach the specific mathematics topics at the elementary level or slightly beyond:

- In at least three-quarters of the districts, less than half of first through third-grade teachers considered themselves "very well-prepared" to teach more than 60% of the topics. A little more than half of teachers in all districts felt "very well-prepared" to teach just three topics (out of 28).
- In at least three-quarters of the districts, less than half of fourth and fifth-grade teachers

[21] Campbell, P. F., Nishio, M., Smith, T. M., Clark, L. M., Conant, D. L., Rust, A. H., DePiper, J. N., Frank, T. J., Griffin, M. J., & Choi, Y. (2014). The relationship between teachers' mathematical content and pedagogical knowledge, teachers' perceptions, and student achievement. *Journal for Research in Mathematics Education, 45*(4), 419-459; Kukla-Acevedo, S. (2009). Do teacher characteristics matter? New results on the effects of teacher preparation on student achievement. *Economics of Education Review, 28*, 49-57; Hill, H., Rowan, B., & Ball, D. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371-406. Blazar (2015) found a relationship between teachers' mathematical content knowledge and two instructional characteristics: ambitiousness of instruction and frequency of errors and imprecision. While these characteristics also related to student achievement, the study did examine the direct relationship between teachers' math content knowledge and student achievement. Blazar, D. (2015). Effective teaching in elementary mathematics: Identifying classroom practices that support student achievement. *Economics of Education Review, 48*, 16-29; Mapolelo, D. C., & Akinsola, M. K. (2015). Preparation of mathematics teachers: lessons from review of literature on teachers' knowledge, beliefs, and teacher education. *International Journal of Educational Studies*, *2*(1), 01-12; Blazar, D. (2015). Effective teaching in elementary mathematics: Identifying classroom practices that support student achievement. *Economics of Education Review*, *48*, 16-29; Hill, H. C., Charalambous, C. Y., & Chin, M. J. (2019). Teacher characteristics and student learning in mathematics: A comprehensive assessment. *Educational Policy*, *33*(7), 1103-1134.
[22] Shirvani, H. (2015). Pre-Service Elementary Teachers' Mathematics Content Knowledge: A Predictor of Sixth Graders' Mathematics Performance. *International Journal of Instruction*, *8*(1), 133-142.
[23] Gorham Blanco, T., & Chamberlin, S. A. (2019). Pre-service teacher statistical misconceptions during teacher preparation program. *The Mathematics Enthusiast*, *16*(1), 461-484.
[24] Banilower, E. R., Smith, P. S., Malzahn, K. A., Plumley, C. L., Gordon, E. M., & Hayes, M. L. (2018). *Report of the 2018 NSSME+*. Horizon Research, Inc. Retrieved from http://www.horizon-research.com/report-of-the-2018-nssme; Banilower, et al. (2013). *Report of the 2012 National Survey of Science and Mathematics Education.* Horizon Research, Inc. Retrieved November 1, 2018, from http://www.horizon-research.com/2012nssme/wp-content/uploads/2013/02/2012-NSSME-FullReport1.pdf.
[25] Schmidt, W. H., & McKnight, C. (2002). *Inequality for all: The challenge of unequal opportunity in American schools.* New York: Teachers College Press, Columbia University.

considered themselves "very well-prepared" to teach more than 50% of the topics. At least 55% of teachers in all districts only felt "very well-prepared" to teach just three topics.

Most research finds that teacher candidates' mathematics coursework seems to yield benefits for their students. Several studies have demonstrated that teachers deliver stronger lessons on topics that they learned in their teacher preparation programs.[26] A study of teacher preparation programs (both traditional and alternative) in New York City found that math courses correlated with increased student achievement in math during the second year of teaching,[27] and another study found that not only the number of content courses but also the types of courses matters for building candidates' knowledge.[28] However, one study found no correlation between teachers' math education credits and student achievement in math.[29]

How much coursework do elementary teacher candidates need?

Prospective elementary teachers would be more prepared to teach elementary math if they took college mathematics courses that were designed specifically for teachers and that imparted a deep understanding of elementary and middle school mathematics concepts.[30] For example, the Conference Board of the Mathematical Sciences (CBMS)[31] recommended that aspiring elementary teachers also take 12 semester-credit hours in "elementary mathematics content," while the National Council of Teachers of Mathematics (NCTM) recommended that aspiring elementary teachers take at least three college-level mathematics courses in the content essential to elementary grades, in addition to instruction on pedagogy.[32] NCTM categorizes this content into five subject areas: numbers and operations, algebra, geometry, measurement, and data analysis and probability.[33] The Mathematical Education of Teachers II (MET II) study draws from the Common Core State Standards to recommend that elementary teachers be prepared in the domains of counting and cardinality, operations and algebraic thinking, numbers and operations, measurement and data, and geometry,

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[26] Hiebert, J., Berk, D., & Miller, E. (2017). Relationships between mathematics teacher preparation and graduates' analyses of classroom teaching. *The Elementary School Journal*, *117*(4), 687-707; Hiebert, J., Berk, D., Miller, E., Gallivan, H., & Meikle, E. (2019). Relationships between opportunity to learn mathematics in teacher preparation and graduates' knowledge for teaching mathematics. *Journal for Research in Mathematics Education*, *50*(1), 23-50; Morris, A. K., & Hiebert, J. (2017). Effects of teacher preparation courses: Do graduates use what they learned to plan mathematics lessons? *American Educational Research Journal*, *54*(3), 524-567.; Suppa, S., DiNapoli, J., & Mixell, R. (2018). Teacher Preparation" Does" Matter: Relationships between Elementary Mathematics Content Courses and Graduates' Analyses of Teaching. *Mathematics Teacher Education and Development*, *20*(2), 25-57.

[27] Boyd, D. J., Grossman, P. L., Lankford, H., Loeb, S., & Wyckoff, J. (2009). Teacher preparation and student achievement. *Educational Evaluation and Policy Analysis, 31*(4), 416-440. This study notes that its findings may differ from those in Harris & Sass (2011) because the Boyd study looked at "data on the characteristics of programs, courses, and field experiences," while the Harris study used course credit hours and hours of in-service training.

[28] This study found that (a) discrete structure and logic and (b) continuity and functions had the strongest effect on candidates' mathematics content knowledge. Qian, H., & Youngs, P. (2016). The effect of teacher education programs on future elementary mathematics teachers' knowledge: a five-country analysis using TEDS-M data. *Journal of Mathematics Teacher Education*, *19*(4), 371-396.

[29] Harris, D. N., & Sass, T. R. (2011). Teacher training, teacher quality and student achievement. *Journal of Public Economics, 95*, 798-812. Note: This study relates to several NCTQ standards. Although it meets the criteria for strong research, the study's findings run contrary to the conclusions of most strong research in the field.

[30] In this vein, a University of Virginia professor of psychology argued that elementary teachers need to be trained to understand and teach the "conceptual side of math," or else they cannot build a strong math foundation for their young students. Willingham, D. (2013). *What the NY Times doesn't know about math instruction*. Retrieved March 12, 2014 from http://www.danielwillingham.com/daniel-willingham-science-and-education-blog/what-the-ny-times-doesnt-know-about-math-instruction

[31] Beckmann, S., Chazan, D., Cuoco, A., Fennell, F., & Findell, B. (2012). The Mathematical Education of Teachers II. In *Issues in mathematics education/CBMS, Conference Board of the Mathematical Sciences* (Vol. 17, pp. 1-86). Retrieved January 7, 2022 from https://www.cbmsweb.org/archive/MET2/met2.pdf.

[32] National Council of Teachers of Mathematics. (2005). *Highly qualified teachers: A position of the National Council of Teachers of Mathematics (Position Paper).* Retrieved April 2022, from https://web.archive.org/web/20060221055354/http://www.nctm.org/about/pdfs/position/qualified.pdf.

[33] National Council of Teachers of Mathematics. (No Date). *Executive Summary: Principles and standards for school mathematics.* Retrieved April 26, 2021 from https://www.nctm.org/uploadedFiles/Standards_and_Positions/PSSM_ExecutiveSummary.pdf.

as well as connections to mathematics topics typically addressed in the middle grades.[34]

Some research casts doubt on the extent to which current teacher preparation programs adequately meet the mathematics needs of aspiring elementary teachers. A survey of over 400 institutions found that most were not meeting the recommendation that elementary candidates take at least 12 semester credit-hours of mathematics content.[35] In a second study conducted 6 years later with over 400 institutions, the researchers again found that preparation programs continued to fall short of the 12 semester credit-hour goal, but saw some progress in using activity-based approaches in these courses, and an increase in instructors with doctorates in mathematics education or teaching experience (though that experience was much more common at the secondary, not elementary level).[36] Another study found that mathematics content courses were inconsistent in whether they engaged teacher candidates in the Common Core Standards for Mathematical Practice.[37]

What types of math courses should elementary teacher candidates take?

The preponderance of available research studies indicates that the mathematics content coursework needed by elementary teachers is neither pure mathematics nor pure methods but a combination of both. This mix of coursework imparts the foundational knowledge of elementary mathematics topics and is a bridge to instruction in the elementary classroom.[38] Teachers with more specialized content knowledge can better design lessons utilizing math-science integration, use manipulatives in their lessons, and employ student-centered approaches to teaching mathematics.[39]

Experts suggest that educator preparation programs should structure requirements to address both subject matter knowledge (including common content knowledge and specialized content knowledge) and pedagogical content knowledge (including knowledge of content and students and knowledge of content and teaching).[40] The approach described by Lee Shulman[41] — built off of early work begun by John Dewey and expanded by Deborah Ball — explains the complexity of teaching by delineating the domains of knowledge needed for teaching.

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[34] Beckmann, S., Chazan, D., Cuoco, A., Fennell, F., & Findell, B. (2012). The Mathematical Education of Teachers II. In *Issues in mathematics education/CBMS, Conference Board of the Mathematical Sciences* (Vol. 17, pp. 1-86). Retrieved January 7, 2022 from https://www.cbmsweb.org/archive/MET2/met2.pdf.
[35] Masingila, J. O., Olanoff, D. E., & Kwaka, D. K. (2012). Who teaches mathematics content courses for prospective elementary teachers in the United States? Results of a national survey. *Journal of Mathematics Teacher Education*, *15*(5), 347-358.
[36] Masingila, J. O., & Olanoff, D. (2021). Who teaches mathematics content courses for prospective elementary teachers in the USA? Results of a second national survey. *Journal of Mathematics Teacher Education*, 1-17.
[37] Max, B., & Welder, R. M. (2019). Engaging prospective elementary teachers in standards for mathematical practice within content courses for teachers. In *Proceedings of the 41st Annual Conference of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1141-1145).
[38] Greenberg, J., & Walsh, K. (2008). *No Common Denominator: The Preparation of Elementary Teachers in Mathematics by America's Education Schools*. National Council on Teacher Quality. However, a recent international comparative study claims that while both content knowledge and pedagogical content knowledge are important, "...CK may not necessarily need to develop before PCK...future reforms should consider the best ways to foster content and pedagogical content knowledge as distinct constructs rather than working under the assumption that developing CK is a necessary prerequisite for developing PCK." (p 16). Murray, E., Durkin, K., Chao, T., Star, J. R., & Vig, R. (2018). Exploring Connections between Content Knowledge, Pedagogical Content Knowledge, and the Opportunities to Learn Mathematics: Findings from the TEDS-M Dataset. *Mathematics Teacher Education and Development*, *20*(1), 4-22.
[39] An, S. A. (2017). Preservice teachers' knowledge of interdisciplinary pedagogy: the case of elementary mathematics–science integrated lessons. *ZDM*, *49*(2), 237-248; Greenstein, S., & Seventko, J. (2017). Mathematical Making in Teacher Preparation: What Knowledge Is Brought to Bear?. *North American Chapter of the International Group for the Psychology of Mathematics Education*; Son, J. W. (2016). Preservice teachers' response and feedback type to correct and incorrect student-invented strategies for subtracting whole numbers. *The Journal of Mathematical Behavior*, *42*, 49-68.
[40] Loewenberg Ball, D., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*(5), 389-407.
[41] Shulman, L. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, *57*(1), 1-23.

What should elementary teacher candidates learn about specialized mathematics content knowledge?

Elementary teachers need to grasp more than the mathematical knowledge and skills required in the curriculum for students. They need to master the mathematical knowledge that is unique to teaching, the "unpacked mathematical knowledge" needed by teachers to make "content visible to and learnable by students."[42] Examples of what teachers need to learn include being able to create and tailor representations of math problems to suit the "instructional purposes," being able to not only carry out but also explain algorithms for solving problems, and conducting error analysis.[43] Teachers may learn more when they "engage with mathematics in ways that afford them more opportunities to explore and link alternative representations, to provide and interpret explanations, and to delve into meanings and connections among ideas."[44] An effort to "unpack the mathematical work of teaching framework" to further explore what teachers should be able to do include activities such as, "Given conflicting explanations, determine which is valid and why," "Write a mathematically valid explanation for a process or concept," "Given a word problem, choose another word problem with the same structure," and "Given a set of representations, choose which does or does not show a particular idea."[45]

What should elementary teacher candidates learn about mathematics pedagogy?

Research on mathematics methods or pedagogy, although limited, also indicates the value of mathematics methods courses,[46] including documented gains in mathematical knowledge for teaching.[47] Research generally supports the importance of teachers' knowledge of fundamental math concepts as well as their ability to apply mathematics content in teaching (learned in mathematics methods courses), rather than their just knowing the mathematics content.[48] In some cases, teacher candidates may need to unlearn some approaches to teaching that they learned through their own schooling.[49] Teacher candidates may also benefit from engaging in field experiences combined with other mathematical preparation.[50]

[42] p. 400. Loewenberg Ball, D., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special?. *Journal of Teacher Education*, *59*(5), 389-407.
[43] Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide?. *American Educator*.
[44] Hill, H. C., & Ball, D. L. (2004). Learning mathematics for teaching: Results from California's mathematics professional development institutes. *Journal for Research in Mathematics Education*, *35*(5), 330-351.
[45] Selling, S. K., Garcia, N., & Ball, D. L. (2016). What does it take to develop assessments of mathematical knowledge for teaching?: Unpacking the mathematical work of teaching. *The Mathematics Enthusiast*, *13*(1), 35-51.
[46] Promoting Rigorous Outcomes in Mathematics and Science Education (2006, December). Knowing mathematics: What we can learn from teachers (Research Report, Vol. 2). East Lansing, MI: Michigan State University; Cavanna, J. M., Drake, C., & Pak, B. (2017). Exploring Elementary Mathematics Teachers' Opportunities to Learn to Teach. *North American Chapter of the International Group for the Psychology of Mathematics Education*; Santagata, R., Yeh, C., & Mercado, J. (2018). Preparing elementary school teachers to learn from teaching: A comparison of two approaches to mathematics methods instruction. *Journal of the Learning Sciences*, *27*(3), 474-516; Giles, R. M., Byrd, K. O., & Bendolph, A. (2016). An investigation of elementary preservice teachers' self-efficacy for teaching mathematics. *Cogent Education*, *3*(1), 1160523.
[47] Laursen, S. L., Hassi, M. L., & Hough, S. (2016). Implementation and outcomes of inquiry-based learning in mathematics content courses for pre-service teachers. *International Journal of Mathematical Education in Science and Technology*, *47*(2), 256-275.
[48] Ball, D., Lubienski, S., & Mewborn, D. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), Handbook on research on teaching (4th ed.). Washington, DC: American Educational Research Association; Guyton, E., & Farokhi, E. (1987). Relationships among academic performance, basic skills, subject matter knowledge, and teaching skills of teacher education graduates. *Journal of Teacher Education, 38*, N5.
[49] Shaughnessy, M., & Boerst, T. A. (2018). Uncovering the skills that preservice teachers bring to teacher education: The practice of eliciting a student's thinking. *Journal of Teacher Education*, *69*(1), 40-55.
[50] Strawhecker, J. (2005). Preparing elementary teachers to teach mathematics: How field experiences impact pedagogical content

Teacher preparation programs in high-achieving nations frequently ensure that teachers not only know the content but also can communicate it. Mathematics-specific pedagogy is part of the preparation of mathematics teachers around the world.[51] An analysis of courses taken by elementary teachers across 17 countries found five commonly required mathematics courses and another seven common mathematics electives addressing both content and pedagogy.[52]

Conclusion

In 2022, NCTQ revised its Elementary Mathematics standard in keeping with the research that elementary teachers must be equipped with the mathematics content and pedagogical knowledge to effectively support learning by all students. NCTQ's examination of the opportunities a program provides to teacher candidates includes the domains of knowledge a teacher needs to bring to the classroom – the ***specialized mathematical content knowledge*** or knowledge of mathematics that is specific to teaching mathematics and p***edagogical knowledge for teaching mathematics*** or the intersections between the mathematical content that comprises the curricula and knowledge of how students learn and effective methods to teach.[53]

NCTQ gathered information on required mathematics courses for elementary teacher candidates to evaluate the time courses dedicate to addressing mathematics topics – Numbers & Operations, Algebraic Thinking, Geometry & Measurement, Data Analysis & Probability, and Mathematics Pedagogy – identified by the MET II research, highlighted in NCTM guidelines, and included in CCSS-M elementary student standards.

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knowledge. *Issues in the Undergraduate Mathematics Preparation of School Teachers*, *4*.
[51] Jensen, B., Roberts-Hull, K., Magee, J., & Ginnivan, L. (2016). *Not so elementary: Primary school teacher quality in high-performing systems*. Washington, DC: National Center on Education and the Economy. Communications with Mdm. Low Khah Gek, Deputy Director, Sciences, Curriculum Planning and Development Division, Ministry of Education, Singapore, March 2008.
[52] Schmidt, W., Burroughs, N., Cogan, L. (2013). *World class standards for preparing teachers of mathematics (Working Paper).* East Lansing, MI: Michigan State University Center for the Study of Curriculum and The Education Policy Center.
[53] Loewenberg Ball, D., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*(5), 389-407.

Appendix B:
Examples of Course Description and Course Syllabus Coding

Coding process

In addition to evaluating course descriptions for all courses, analysts also reviewed all available syllabi. Based on the different levels of detail found in course descriptions and syllabi, parallel coding systems were developed that share common outputs, but allow for the most granular level of analysis possible under each data source. It's important to note that not all syllabi include a detailed course schedule with course lecture/discussion topics. In these "less detailed" cases, if the syllabus provided information that extended beyond the course description, the document was still used, but the coding process aligned with the course description approach.

Analysts were randomly assigned to programs and reviewed all courses identified in that program.

Under the course description approach detailed above, the percentage of a course dedicated to the topics was calculated by counting the number of references to each of the seven topics (the four content topics, mathematics pedagogy, other content, and other pedagogy) and dividing those figures by the total number of references in the description (the "reference count"). Under the syllabus approach, the topic percentages are determined by first calculating the percentage of each topic covered in each "unit" (e.g., course meeting) and then summing those results to determine the overall instructional time the course dedicates to each topic.

Figure 4. Basic Coding structure

<table>
<tr><th colspan="2">When coding a **Course Description**</th><th colspan="2">When coding a **Syllabus**</th></tr>
<tr><td rowspan="3">Reference Count</td><td rowspan="3">Count of the total number of references to each identifiable topic (e.g., Numbers & Operations)</td><td>Unit Type</td><td>Identify the type of calendar (daily, weekly, etc.) used in the syllabus</td></tr>
<tr><td>Unit Count</td><td>Identify the total number of units (days, weeks, etc.) that are defined in the course schedule.</td></tr>
<tr><td>Reference Count</td><td>For **each unit**, count of the total number of references to each identifiable topic</td></tr>
<tr><th colspan="2">Within the course description or details presented in the syllabus…</th><th colspan="2">Within **each unit specified in the syllabus**…</th></tr>
<tr><td>1. Numbers & Operations</td><td colspan="3">Count of the number of references to Numbers & Operations including any subtopics within that domain</td></tr>
<tr><td>2. Algebraic Thinking</td><td colspan="3">Count of the number of references to Algebraic Thinking, including any subtopics with that domain</td></tr>
<tr><td>3. Geometry & Measurement</td><td colspan="3">Count of the number of references to Geometry & Measurement including any subtopics with that domain</td></tr>
<tr><td>4. Data Analysis & Probability</td><td colspan="3">Count of the number of references to Data Analysis & Probability, including any subtopics with that domain</td></tr>
<tr><td>5. Mathematics Pedagogy</td><td colspan="3">Count of the number of references to mathematics instructional approaches</td></tr>
<tr><td>6. Other Content Topics[54]</td><td colspan="3">Count of the number of references that address mathematics content topics outside any of the four content areas (e.g., references to trigonometry or calculus)</td></tr>
<tr><td>7. Other Pedagogy Topics[53]</td><td colspan="3">Count the number of references that address general pedagogy or subject-specific pedagogy for subjects other than mathematics.</td></tr>
</table>

Example of course description analysis

- Based on the full description, the analyst counts the total number of references that fall under each of the seven topics (the four content topics, mathematics pedagogy, other content topics, and other pedagogy)
- The analyst then categorizes each of the references under the appropriate topic

EXAMPLE Course description: *The focus of this course is on topics of mathematics in the primary grades and how these concepts and skills connect to mathematics on the horizon (grades 3-8 and beyond). The course will focus on topics in the domains of counting and cardinality, numbers and operations, algebraic thinking, measurement, and geometry and connections among concepts.*

[54] While coded, “Other Content Topics” and “Other Pedagogy Topics” were not used to evaluate the Elementary Mathematics Standard.

Reference Count	Numbers & Operations	Algebraic Thinking	Geometry & Measurement	Data Analysis & Probability	Other Topics	Math Pedagogy	Other Pedagogy
7	4	1	2	0	0	0	0

- Using the individual topic counts as numerators and the reference count as the denominator an initial percentage of time dedicated to each topic is automatically calculated in the database

Numbers & Operations	Algebraic Thinking	Geometry & Measurement	Data Analysis & Probability	Other Topics	Math Pedagogy	Other Pedagogy
57%	14%	29%	0%	0%	0%	0%

- The resulting percentages are used in combination with the course instructional hour count to estimate the number of instructional hours dedicated to each topic. (This step is identical for course description and syllabus analysis and is described in more detail below.)

Example of detailed syllabus analysis

- Based on the course schedule, the analyst determines the number of defined units (e.g., class meetings or weekly topics) in the syllabus. Course schedules are most often either daily or weekly. For this example, this is a course with a weekly schedule that includes 14 defined units.
- The Reference Count and topics addressed within each unit are categorized.

Unit Count	14							
Unit #	Reference Count	Numbers & Operations	Algebraic Thinking	Geometry & Measurement	Data Analysis & Probability	Other Topics	Math Pedagogy	Other Pedagogy
1	2				2			
2	2				2			
3	1				1			
4	1				1			
5	1			1				
6	1			1				
7	2			2				
8	2		1	1				
9	1		1					
10	1		1					
11	3	2	1					
12	1	1						
13	1	1						
14	1	1						

- The following calculations of the percent of each unit dedicated to each topic are automatically generated based on those data. The sum of units devoted to each topic is divided by the total number of units (14) to calculate the percentage of course time devoted to each topic ("course coverage").

Unit #	Numbers & Operations	Algebraic Thinking	Geometry & Measurement	Data Analysis & Probability	Other Topics	Mathematics Pedagogy	Other Pedagogy
1	0%	0%	0%	100%	0%	0%	0%
2	0%	0%	0%	100%	0%	0%	0%
3	0%	0%	0%	100%	0%	0%	0%
4	0%	0%	0%	100%	0%	0%	0%
5	0%	0%	100%	0%	0%	0%	0%
6	0%	0%	100%	0%	0%	0%	0%
7	0%	0%	100%	0%	0%	0%	0%
8	0%	50%	50%	0%	0%	0%	0%
9	0%	100%	0%	0%	0%	0%	0%
10	0%	100%	0%	0%	0%	0%	0%
11	67%	33%	0%	0%	0%	0%	0%
12	100%	0%	0%	0%	0%	0%	0%
13	100%	0%	0%	0%	0%	0%	0%
14	100%	0%	0%	0%	0%	0%	0%
Sum	3.7	2.8	3.5	4	0	0	0
Course Coverage	26%	20%	25%	29%	0%	0%	0%

The final step in both course description and syllabus analysis is to convert percentages into instructional hours. This calculation first adjusts for non-semester-based academic calendars). For example, the number of instructional hours in a 3-credit course at an institution that uses quarters is the equivalent of just 2 semester credit hours.

Following any necessary calendar-based adjustments, the number of semester credit hours is multiplied by 15 instructional hours to obtain the total instructional hours for the course (a standard 3 semester credit hour course is 45 instructional hours). All institutions make the calendar type and credit counts publicly available. This information has been collected since the foundation of the *Teacher Prep Review* and used in calculations under multiple standards, including Elementary Mathematics in the past.

Once the total instructional hours have been calculated for each course, the resulting figure is multiplied by the topic percentages to produce the estimated number of instructional hours under each topic. Using the syllabus example above, which is a 4-semester credit hour course,[55] we would estimate:

- Numbers & Operations: 16 instructional hours (60 x 0.26)
- Algebraic Thinking: 12 instructional hours (60 x 0.20)
- Geometry & Measurement: 15 instructional hours (60 x 0.25)
- Data Analysis & Probability: 17 instructional hours (60 x 0.29)

[55] A 4-credit course represents 60 instructional hours.