to Standard 5

Prep Resources: Teaching Elementary Math Content

Louisiana State University and Agricultural & Mechanical College has generously provided syllabi for three courses that address the critical subject areas of numbers and operations, geometry, algebra and data analysis.

The syllabi for the three courses follow this cover page:

- Math 1201 Number Sense and Open-Ended Problem-Solving
- Math 1202 Geometry, Reasoning and Measurement
- Math 2203 Proportional and Algebraic Reasoning

Following the syllabi are lecture notes for Math 1201 Number Sense and Open-Ended Problem-Solving.

National Council on Teacher Quality

1120 G Street, NW, Suite 800
Washington, D.C. 20005
Tel: 202 393-0020 Fax: 202 393-0095
Web: www.nctq.org

Louisiana State University and Agricultural & Mechanical College

Math 1201, [redacted] Fall, 2011

[redacted]

Office: [redacted]
Email:
Phone:
Office hours:

Prerequisites: None.

Goals: This course is a *mathematics course for teachers* focusing on elementary school mathematics from the perspective of *teaching* these concepts to elementary school students. The main goal, of course, is to acquire a solid knowledge of the material. But an elementary school mathematics teacher needs to know much more, including:

(i) how YOU **present the material** in the simplest, clearest way;
(ii) how YOU recognize the **appropriate sequential orde**r for developing mathematics skills;
(iii) how to i**dentify and alleviate** "pot-holes," i.e. what the students will find difficult and what errors they are likely to make, and
(iv) how YOU explain that each topic helps **advance the mathematical level** of the students.

Texts:

• *Elementary Mathematics for Teachers,* by Thomas H. Parker and Scott Baldridge.

• *Primary Mathematics Textbooks (U.S. Edition)* — Primary Mathematics 3A, 4A, 5A, and 6A and Workbook 5A. Students using this elementary school curriculum were tested as the best in the world. We will do many problems from these books.

These books will also be extraordinarily useful as you begin your teaching career.

Expectations: Classes will be primarily lectures, with problem solving individually and in small groups. You are expected to take complete notes, to participate in class activities, and to ask questions about what you do not understand. We will **take the daily attendance** by homework or quizzes or special assignments.

Grading Policy: There will be 3 hourly exams, a final exam, and homework/quizzes/special assignments, with percentages as shown below. Missed exams will count as 0 points. Only under rare circumstances (such as illness with a doctor's written excuse) will a make-up exam be given.
There is NO MAKE-UP on your daily grades.

The grading scale is straightforward:

90% - 100% = A	77% - 79% = B-	60% - 66% = D
87% - 89% = A-	74% - 76% = C+	0% - 59% = F
84% - 86% = B+	70% - 73% = C	
80% - 83% = B	67% - 69% = C-	

This grading scale will not be curved, even at the end of the semester. All grades are based on how well *each student learns the material*, so grades are not competitive. We could have an “all – A” class, or a “no – A” class, or any combination otherwise.

Tentative Exam Schedule:

15 % First Hourly Exam		[redacted]
15 % Second Hourly Exam		
15 % Third Hourly Exam		
30 % Final Exam		TBA
25% Homework & Quizzes Daily		

Homework and Quizzes: Mathematics is learned by practice, and confidence is gained through mastery of the material. Homework will be assigned daily in class and is due at the beginning of the next class. There might be occasional quizzes. Homework and quizzes will count 25% of your grade. This will be done according to the following point system: over the semester 250–300 points will be given in graded homework and quizzes. **If you earn 200 points you will get the full 25%**, otherwise your score will be proportional to how you did out of 200 points. It will take your consistent effort on these assignments to earn 200 points.

You should plan on spending at least 2 hours of homework for each class meeting. It is essential not to get behind; we will work at a brisk pace.

You are encouraged to work in groups on difficult homework problems. This doesn’t mean that you should copy from someone or allow someone to copy from you. Rather, you should explain the difficult problems and solutions to one another to help each other understand.

Do not let yourself get behind the class! As in most math courses, the material progressively builds upon itself. If you do not understand a particular topic ask in class or in office hours.

Calculators: Calculators will not be used for this class, and will not be allowed for exams.
A successful *elementary school teacher* should be confident and comfortable solving problems mentally and on paper. One of the goals of this course is to develop that facility.

Important dates:

- Dates for dropping or withdrawing from a course, see calendar of semester.
- Monday, September 5, 2011 – holiday, no class.
- Friday, Oct 14, 2011 – holiday, no class.
- Wednesday and Friday, November 23 & 25, 2011 Thanksgiving holiday, no class.
- December 7, 2011 Last Day of Math 1201 – 2

GOOD LUCK, STUDY HARD, LET’S HAVE FUN.

MATH 1201

DAILY SCHEDULE

DATE	BRING TO CLASS	SECTION IN CLASS	READ SECTION(S), WORK HOMEWORK
	text[1]	1.1	Preface, HW Set 1
	text	1.2	1.2, HW Set 2
	text	1.3, 2.1	1.3 & 2.1, Set 3
	text	1.4	1.4, Set 4
	text	1.5	1.5, Set 5
	text, 3A	1.6, 1.7	1.6 & 1.7, Set 6 (Set 7, 1,2,4,5)
	text, 3A, 5A	2.1, 2.2	All Chap 2, Set 7, 1,2,3,5; Set 8, 1,2
	text, 3A, 5A	2.2, 2.3	Intro Chap 3, Finish Set 8, Set 9
	text, 3A	3.1	3.1, Set 10
	text, 3A	3.2, 3.3	3.2, 3.3, Set 11
	text, 3A	3.3	3.3, Set 12, not #7, Memorize sqrs to 20
	text, 3A	3.4	3.4, Set 13
	text, 4A, WB5A	3.5 est.	3.5, Set 14, 1-3,6c: wkbk 5A, ex 4, 1-4; 5abef, 6abef
	text, 5A & WB5A	3.6 L Div.	3.6, Set 15; wkbk 5A, Ex 6 (p.16)
		EXAM 1	
	text, 6A	4.1	4.1, probs 3,5,6 of Set 16
	text, 6A	4.1	Finish Set 16
	text	4.2	4.2, Set 17
	text	4.3	4.3, Set 18
	text, 4A	5.1	5.1, Set 19

[1] ALWAYS BRING THE MAIN TEXT and CLASS STUDENT BOOKS 3a-6a AND 5 WKBK, YOUR MATH NOTEBOOK (LOOSELEAF PAPER) AND HOMEWORK.

MATH 1201 Spring 2012 DAILY SCHEDULE

DATE	BRING TO CLASS	SECTION IN CLASS	READ SECTION(S), WORK HOMEWORK
	text	5.2,5.3	5.2, 5.3, Set 20, Set 21
	text	5.4	5.4, Set 22
	text, 3A, 4A	6.1	6.1, Set 24
	text, 4A, 5A	6.2	6.2, Set 25
	text, 5A	6.3	6.3, Set 26
	text, 5A	6.4	6.4, Set 27
	text	6.5	6.5, Set 28
		EXAM 2	
	text	6.6	6.6, Set 29
	text, 5A, 5AWB, 6A	7.1, 7.2	7.1, 7.2 Set 30
	text, 6A	7.2	7.2, Set 31
	text, 6A	7.3	7.3, Set 32
	text	8.1	8.1, Set 34
	text	8.2	8.2, Set 35
	text	8.3	8.3, Set 36
	text, 4A, 5A	9.1	9.1, Set 37
		EXAM 3	
	text, 4A, 5A	9.2	9.2, Set 38
	text	9.3	9.3, Set 39
	text	9.4	9.4, Set 40
	REVIEW MATERIAL FOR FINAL EXAM		
_______		**FINAL EXAM**	

Louisiana State University and Agricultural & Mechanical College

MATH 1202 Spring, 2012

Instructor: [redacted]

Office:
Email:
Office hours: [redacted]

Prerequisites: 1201.

Goals: This course is a mathematics course focusing on elementary school mathematics. The main goal, of course, is to acquire a solid knowledge of that material. But an elementary school mathematics teacher needs to know much more, including: (i) how to present the material in the simplest, clearest way, (ii) the appropriate sequential order for developing mathematics skills, and (iii) what the students will find difficult and what errors they are likely to make, and (iv) how each topic helps advance the mathematical level of the students.

Texts:

- Elementary Geometry for Teachers, by Thomas H. Parker and Scott Baldridge.
- Primary Mathematics textbooks (U.S. Edition) — Primary Mathematics 3B, 4A, 5A, 5B and 6B. Students using this elementary school curriculum were tested as the best in the world. We will do many problems from these books.
- New Elementary Mathematics 1
- Manipulative kit (optional, but recommended)
- In addition, you will need items including: ruler, protractor, compass (for drawing circles, not navigation), scientific calculator.

Expectations: Classes will be primarily lectures, with problem solving individually and in small groups. You are expected to take complete notes, to participate in class activities, and to ask questions about what you do not understand. Attendance will be taken.

Grading Policy: There will be 4 hourly exams, a final exam, homework/quizzes, as well as a project (see below) with percentages as shown below. Missed exams will count as 0 points. Only under rare circumstances (such as illness with a doctor's written excuse) will a make-up exam be given. However, in such a case, do not wait until you return to class to speak with me about a make-up. Either arrange for it beforehand, or email me no later than the day of the missed test.

Grade Breakdown:	45%	4 Exams
	25%	Final Exam
	20%	Homework & Quizzes
	10%	Project

The grading scale is straightforward:

90% − 100%	=	A	80% − 90%	=	B
70% − 80%	=	C	60% − 70%	=	D
0% − 59%	=	F			

This grading scale will not be curved, even at the end of the semester. All grades are based on how well each student learns the material, so grades are not competitive. Grades in 1202 are based on understanding, not upon comparisons with other students.

Homework and Quizzes: Mathematics is learned by practice, and confidence is gained through mastery of the material. Homework will be assigned daily in class and is due at the beginning of the next class. There will also be frequent quizzes. Homework and quizzes will count 25 % of your grade. This will be done according to the following point system: over the semester 250–300 points will be given in graded homework and quizzes. If you earn 200 points you will get the full 20%, otherwise your score will be proportional to how you did out of 200 points. It will take consistent effort on these assignments to earn 200 points.

You should plan on spending at least 2 to 3 hours on homework for each class meeting. It is essential not to get behind; we will work at a brisk pace.

You are encouraged to work in groups on difficult homework problems. This doesn't mean that you should copy from someone or allow someone to copy from you. Rather, you should explain the difficult problems and solutions to one another to help each other understand.

Do not let yourself get behind the class! As in most math courses, the material progressively builds upon itself. If you do not understand a particular topic ask in class or in office hours.

The Project: The project for this course is a service-learning project. Each student will be required to visit a local elementary school 6-7 times during the semester to tutor math. A journal will be required for each visit as well as a final paper on the experience. Final project due within 2 weeks of final visit to the school, but no later than Wed Apr 4. Intermediate due dates are explained in the project guidelines and must be met, even if it means rescheduling visits. No late projects will be accepted.

Calculators: Calculators may be used for this class, and will be allowed for exams. However, no graphing calculators will be allowed during exams!

Other Miscellany: I will use Moodle to post handouts, grades, and communicate via email. Be sure that you check your PAWS email every day (or have it forwarded to your preferred email address). Several handouts have been posted on Moodle, and will be required for homework. You should download, print and place these into your class binder immediately.

Important dates:

- For holidays/university closures, see http://appl003.lsu.edu/slas/registrar.nsf/$Content/Academic+Calendars.
- Jan 24th: Last day to drop without a W
- Jan 26th: Last day to add
- April 2nd: Last day to drop or arrange for conflicts with final exam
- Final Exam: Wednesday, May 9th, 5:30-7:30 pm

MATH 1202 *tentative* **SCHEDULE and ASSIGNMENTS** **Spring 2012**

EGT: Elementary Geometry for Teachers; NEM: New Elementary Math 1 Syllabus D; BH: Big Handout; Primary Math Books 3B, 4A, 5A, 5B, 6B

	Points, lines, planes; Units of length; Triangle inequality	**EGT:** Read 1.1; HW Set 1 # 2, 3, 7, 9(lines should be drawn through *pairs* of points) Read 1.2; HW Set 2 # 7, 10, 11, 12, 13, 14	**3B**: p. 18 # 1a, 2c, 3ac, 4d**;** p. 23 # 1a, 2c, 3cf, 4cg; p. 28 # 1a, 2b, 3c, 4b, 6ce, 7ce	**5A**: p. 46 # 6bd, 9agi; p. 95 # 22ad, 23ad, 31	**5B**: p. 21 # 9d, 12aef, 13
	Units of weight and capacity	**EGT**: Read 1.3	**3B**: p. 33 # 1e, 2c, 3c, 4ce; p. 42 # 1a, 2c, 3ac; p. 51 # 3abc, 4cg; p. 56 # 4abc, 5abc	**5A**: p. 46 # 6ce, 9b; p. 95 # 22bc, 23bc	**5B**: p. 21 # 9a, 10de, 11ef, 14
	Angles	**EGT**: Read 1.4; HW Set 4 # 8; Read 2.1 p. 27-29; HW Set 5 # 2, 8, 9	**3B**: Read p. 92-95 **4A**: Read p. 74-77	**5A**: p. 84 #1, 2ab, 3ab p. 88 # 4, 5, 6, 7 **BH**: p. 1	**NEM**: p. 237 #2bc, 3ab; p. 240 # 1ac, 2cf
	Parallel and perpendicular lines; sum of angles in a triangle; exterior angles isosceles triangles	**EGT**: Read 2.1 p. 29-34; HW Set 5 # 10, 12, 13 Read 2.2; HW Set 6 # 3, 4, 5, 6, 7; Read 2.3; HW Set 7 # 2, 3, 4	**5B**: Read p. 57-67 and answer p. 58 #3, p. 59 # 6; p. 60 #9; p. 62 #3; p. 63 #4; p. 64 # 8; p. 67 #3abd	**NEM**: p. 264 #2, 3hi; p. 268 #1def, 2ab, 3a	
	Symmetry	**NEM**: p. 291 # 1, 2bdf; p. 295 # 1, 2, 3,5	**BH**: p. 2, 3		
	Quadrilaterals and their angles	**EGT**: Read 2.4; HW Set 8 # 2, 3, 9, 10, 11	**5B**: Read p. 68-75; Answer p. 70 #3; p. 71 #4, 6; p. 75 #3, 5	**6B**: Read p. 62-65; Answer p. 66 # 1, 4, 5; p. 67 all	
	Constructions	**EGT**: Read 2.5. Omit constructions 6, 7.	**EGT:** HW Set 9 # 2, 3, 4, 6ab, 8 (trace it; don't cut out)	**BH**: p. 4	Construct an isosceles triangle.
	Contructions	**Practice** all constructions assigned 9/1.			
	Unknown Angle Problems	**EGT**: Read 3.1. **BH**: p. 5	**6B**: Write teacher's solutions: p. 70 # 26; p. 71 # 27, 28; p. 76 # 34; p.109 # 35	**NEM:** Write teacher's solutions: p. 264 #3j	
	Unknown Angle Problems	**6B**: Write teacher's solutions: p.109 # 36, 37; p.115 # 44	**NEM:** Write teacher's solutions: p. 264 #4hl; p. 268 # 3b		
	Parallel lines conjecture	**EGT**: Read 3.2; HW Set 11 # 1c, 2bc	**NEM:** p. 248 # 2, 7, 8eh; p. 264 # 4n (No Teacher's Solns required)		
	Parallel lines conjecture and its converse	**NEM**: Write teacher's solutions: p. 250 # 8ab, 9b, 10d; p.264 # 4dk	**6B**: p. 82 #40, p. 116 # 45 Write teacher's solutions.	**BH**: p. 6 # 1, 2, 3ab (No teacher's soln for #3.)	
	Polygons; sums of angles; regular polygons	**EGT**: Read 3.3; HW Set 12 # 5cd, 6	**NEM**: p. 274 # 1b, 2, 3ac, 4, 5, 6, 10bc, 11	**BH:** p. 6 #3c, p. 7	
	Review				
	Test 1				

	Unknown angle proofs	**EGT:** Read 4.1; HW Set 13 #1, 5, 7, 8, 9, 10, 12		
	Congruent Triangles	**EGT:** Read 4.2; HW Set 14 # 1, 3, 6, 7, 8, 11		
	Proofs Using Congruent Triangles	**EGT**: Read 4.3; HW Set 15 # 1, 2, 3, 4, 6, 7, 8, 12	**EGT:** Learn p. 99	
	Using Congruent Triangles; Properties of Quads	**EGT**: Read 4.4 ; HW Set 16 # 3	**BH**: p. 8 #1-9	**NEM**: p. 281 # 1, 2, 3 (just find x and y; no proofs)
	Using Congruent Triangles; Properties of Quads	**EGT**: HW Set 16 # 5, 6	**BH**: p. 8 #10-17	**BH:** p. 9
	Banquet Tables BH pgs 10-12	**BH:** p. 13,14,15	**3B:** p. 103 # 5	
	Area and perimeter of rectangles; altitudes	**EGT**: Read 5.1, 5.2	**BH:** p. 16 **3B:** p. 99 # 5	**4A:** p. 91-93 # 1ac, 2ac, 3, 4, 6, 9
	Area of triangles, trapezoids, parallelograms	**EGT**: Read 5.3 **BH:** p. 17 #3,4	**5A:** p. 68 # 3; p. 69 # 4, 5ac; p. 70 all	**NEM:** p. 336 # 1, 3, 5, 6, 8, 10, 12, 13, 14, 15, 16, 18ace
	Area applications	**EGT:** HW Set 20 # 12, 14ab	**BH:** 18	**NEM:** p. 344 # 1b, 2, 3, 4, 6, 12, 14, 21
	Review			
	Test 2			

Mar 12	*Pythagorean Theorem and Square Roots*	**EGT:** Read 6.1; HW Set 21 # 1, 2, 4, 5, 7ac, 8, 9, 10, 11 Read 6.2; HW Set 22 # 1, 2abc, 3, 4, 8, 10, 13, 14		
Mar 14	*Special Triangles*	**EGT**: Read 6.3; HW Set 23 # 1, 2abc (30° angle is opposite y), 3, 4, 5, 6, 7, 8, 10(Note: correction RS=12 not QR=12), 11, 13, 14, 16 Note: Give ***exact, simplified*** answers to ***all*** problems regardless of book's instructions.	**BH**: p. 17 # 1, 2, 5; pg. 19	
Mar 16	*Similarity; Similar Triangles*	**EGT**: Read 7.1 ; HW Set 24 # 6, 8 Read 7.2; HW Set 25 # 2, 3, 5	**BH**: p. 20,21	**NEM:** p. 378 # 3abc, 4ac, 7 p. 390 # 1, 2, 3, 8
Mar 19	*Proving Similar Triangles*	**EGT**: HW Set 24 # 9, 10, 11, 12, 13, 14 HW Set 25 # 4 (Hint: only 1 pair of similar triangles in b), 7, 8, 9 Note: Do not round. Give answers as reduced improper fractions where necessary.	**BH**: p. 22	
Mar 21	*Scaling areas/similarity* *BH pgs. 23-25*	**EGT*:** Read 8.1; HW Set 28 # 7, 8, 9, 10, 11 Note: 9b should read 'What is the ratio AC:CD?', and #11 should read 'AB is parallel to DE'. Note: Do not round. Give answers as reduced improper fractions where necessary.	**BH**: p. 26	
Mar 23	*Circles: Arclength, and Circumference* *BH pg. 27*	**EGT**: Read 8.2; HW Set 29 # 4 (exact answer), 7 **6B:** p. 30	**BH**: p. 28 #1 (arclength), 2, 4 (perimeter)	**NEM:** p. 341 # 1c (circumference), 2a (perimeter), 3a, 7
Mar 26	*Circles: Area and Area of a Sector*	**EGT**: Read 8.3; HW Set 30 # 5 **6B:** p. 36, 37	**BH**: p. 28 #1 (area), 3, 4 (area)	**NEM:** p. 341 # 1c (area), 2a (area), 4a, 5c, 6c
Mar 28	*Applications; converting area units*	**EGT**: HW Set 28 #5(part b should refer back to #14, p. 128), #6 (No teacher's soln. required); HW Set 29 # 9ab (exact answers); HW Set 30 # 3, 4 Read 8.4 p. 188 "Choice of Units"; HW Set 31 #1	**BH**: p. 29,30	**NEM:** p. 345 # 7*, 8, 9, 17, 18, 20 *all units on #7 are m
Mar 30	*Review*			
Apr 2	*Test 3*		**BH**: pg. 31	

Apr 4	*3-D figures; nets*	**NEM:** p. 311 # 1, 2abcde, 6a	**BH:** 32,33	
Apr 16	*Surface area of cylinders, prisms, pyramids*	**EGT**: HW Set 34 # 16acd **BH**: p. 34, p. 43 #1	**NEM:** p. 357 #3(SA only), 4a (exact SA) 5ad (exact SA), 7, 9bc, 10b, 14	
Apr 18	*SA applications; volume of prisms and cylinders*	**EGT**: Read 9.3; HW Set 34 # 2, 3, 4, 5, 6, 7 (exact answer), 8, 10, 11, 12, 13, 14, 16ab	**NEM**: p. 357 #3 (vol), 4 (vol), 5bc (vol), 9a, 10a, 12 **5B**: p. 85	**6B:** p. 59 **BH:** p. 35 #2, 4, 5; p. 36, p. 43 #2A-D
Apr 20	*Volume of cone, pyramid, sphere; conversions*	**EGT**: Read 9.4; HW Set 35 # 2, 3(exact answer), 4(exact answer), 8; Read 9.5; HW Set 36 # 2, 9, 11 (Vol only; ignore the 12cm); HW Set 33 # 1bc, 2, 3, 4ac	**NEM:** p. 358 # 8	**BH**: p. 35 #1, 3, 6, 7, p. 43 #2 E, F
Apr 23	*Volume Applications*	**BH:** p. 37,38		
Apr 25	*Scaling Volume/SA*	**EGT**: HW Set 35 # 1; HW Set 36 # 3, 5	**BH:** p. 39	
Apr 27	*review*			
Apr 30	*Test 4*			
May 2	*Transformations*	**EGT:** Read 4.5	**BH**: p. 40,41,42	
May 4	*Review for final*			
Wed, May 9	***FINAL EXAM 5:30-7:30 PM***	**Comprehensive**		

Louisiana State University and Agricultural & Mechanical College

MATHEMATICS 2203
PROPORTIONAL AND ALGEBRAIC REASONING

Instructor: [redacted] Spring 2012
Office: [redacted] Phone:
E-mail address: Office Hours:

COURSE DESCRIPTION:

MATH 2203: Proportional and Algebraic Reasoning (3 credit hours)
Prerequisites: Professional Practice Block 1; 12 semester hours of mathematics including Math 1201 and 1202; concurrent enrollment in EDCI 3124 and EDCI 3125. 2 hours lecture, 2 hours lab/field experience. Mathematics content course designed to be integrated with Praxis II with the principles and structures applied to mathematical reasoning applied to the grades K-5 classroom. Development of a connected, well balanced view of mathematics; interrelationship of patterns, relations, and functions; applications of proportional and algebraic reasoning in mathematical situations and structures using contextual, numeric, symbolic and graphic representations; written communication of mathematics.

COURSE PURPOSE AND GOALS:
MATH 2203 builds on the foundation of mathematics concepts of problem solving, number and operations, measurement and geometry developed in MATH 1201 and MATH 1202. (Prior completion of these two courses is required.) Concurrent enrollment in EDCI 3124 (Mathematics Theory and Practice in the Elementary Grades) and EDCI 3125 (Elementary and Middle School Science) is required.

The student will:

- increase knowledge, understanding, and application of proportional and algebraic reasoning
- develop the mathematical processes of "finding, describing, explaining, and predicting" through the use of patterns
- use multiple representations (contextual, tabular, numeric, symbolic and graphic) to understand and make connections among mathematical concepts
- understand how math concepts evolve from concrete examples to generalizations expressed by function rules
- understand and analyze change in various contexts
- develop proportional reasoning skills by comparing quantities, looking at relative ways numbers change, and thinking about proportional relationships in linear functions.
- develop conceptual understanding of important mathematical principles, their interrelationship, and their vertical development

ELEMENTARY SCHOOL SITE-BASED MATHEMATICS TUTORING:
MATH 2203 includes required site-based lab/field experience in K-5 mathematics classrooms. This will occur at local public elementary schools. There will be a specific assignment of school, teacher, classroom, days, and times. **LSU now requires travel information from each student for each trip. It is your responsibility to complete the *Service-Learning Student Trip Travel***

***Insurance On-line Form* (*lsu.edu/riskmgt/triptravelservice*) or**
(*lsu.edu/riskmgt/triptravelmobile*) . Additional information about tutoring will be given in class.

CLASS MATERIALS REQUIRED: 2 Composition Books, Colored Pencils

REFERENCES: (You are not required to purchase these.)
Pearson/Allyn and Bacon, Van de Walle (2010). *Elementary and Middle School Mathematics: Teaching Developmentally*
National Council of Teachers of Mathematics (2001). *Navigating through algebra in grades PreK – 2*
National Council of Teachers of Mathematics (2001). *Navigating through algebra in grades 3 – 5*

CONTENT OUTLINE:
I. Algebraic Thinking
II. Patterns, Relationships, Functions
III. Algebraic Symbols and Variables
IV. Mathematical Models
V. Analyzing Change
VI. Proportional Reasoning

GRADING PROCEDURE:
Grading Scale: 90-100% A 80-89.9% B 70-79.9% C 60-69.9% D Below 60 F

Semester grades:	2 tests	40%
	Lab/Field Experience	25%
	To include: Initial Report, Interim Analysis, Final Report, Number of Sessions, Summary Activities, Student's Journal	
	In class written assignments, quizzes	10%
	Final Exam	25%
		100%

There will be an in-class assignment or activity during each class period that is not a test day. These will be graded and one will be dropped at the end of the semester. There will be no make-ups for absences, late arrivals, or early departures. A missed assignment will be recorded as a zero on the assessment. Only partial credit will be given for other categories of assignments/materials that are submitted late.

If you are absent, it is your responsibility to find out what is covered in class and what the assignment is. You are expected to have all work completed when you return to class and be prepared for class. Also, please plan to be here on test days which are in bold print on your syllabus. Tests are extremely difficult to make up. If there is a major emergency and you do miss a test, you will only be allowed to make it up if you contact me by phone or e-mail no later than the day of the test.

ACADEMIC HONESTY:
All students are responsible for adhering to the highest standards of honesty and integrity in every aspect of their academic careers. The penalties for academic dishonesty can be severe and ignorance is not an acceptable defense at Louisiana State University. The LSU Student Code of Conduct can be accessed at
http://appl003.lsu.edu/slas/dos.nsf/$Content/Code+of+Conduct?OpenDocument

CLASS SCHEDULE – SPRING 2012

Please note: Any [redacted] session could have class and lab/field experience interchanged if circumstances warrant. If your lab/field experience is scheduled at a different time, please make sure the [redacted] class times remain reserved for your attendance in class if schedules are changed.

[redacted]			
	Course Introduction What is Algebraic Thinking?		Patterns and Relationships
	Patterns, Relationships and Functions		Patterns, Relationships and Functions **Field Experience Sign-up** **All Sections - 205 Prescott Hall** **Friday, Jan 27 8:30 a.m.**
	Functions and Inverses		Functions and Inverses **Math Tutoring Session Orientation**
	TEST 1		Math Tutoring Session – Week 1
	Symbols and Variables		Math Tutoring Session – Week 2
	MARDI GRAS		Math Tutoring Session – Week 3
	Mathematical Models **Field Experience Initial Report due**		Math Tutoring Session – Week 4
	Solving Equations		Math Tutoring Session – Week 5
	Solving Equations and Systems of Equations		Math Tutoring Session – Week 6
	Solving Equations and Systems of Equations **Field Experience Interim Analysis due** **LEAP TESTING in EBRPSS – Phase 1** **Check with your school to see if you may attend this day.**		Math Tutoring Session – Week 7
	TEST 2		Math Tutoring Session – Week 8
	Analyzing Change **SPRING BREAK -EBRPSS**		**SPRING BREAK -EBRPSS**
	SPRING BREAK - LSU		**SPRING BREAK – LSU** **LEAP TESTING in EBRPSS– Phase 2** **LSU students may not attend public schools.**
	Analyzing Change **LEAP TESTING in EBRPSS– Phase 2** **LSU students may not attend public schools**		Math Tutoring Session – Week 9
	Analyzing Change		Math Tutoring Session – Week 10
	Final Class Summarize		Math Tutoring Session – Makeup Day Field Experience **Final Report Due** **Summary Activities Due**
	FINAL EXAM – GROUP EXAM Wednesday, May 9 – 5:30 pm - 7:30 pm		

What to bring to class:
Ask students to bring PM 4A and 5A.

1.1 Counting

Go over syllabus (10 min – Talk thru)

1. * Taught by a mathematician — I do mathematics and teach it – full time
   * Emphasis on mathematics actually taught in Elem. school.

2. Graded like a math course

3. Go over point system; how to work in groups on HW.

4. Texts: Singapore & Math for Elem. Teachers

Elem. School Math is familiar, but not trivial.

Teacher must know:

* why things are true
* how to explain them in several ways
* pitfalls

Types of elementary questions:

Why is $(-1) \times (-1) = 1$?

How do you show the area of a circle is πr^2?

Make up a word problem for $\frac{3}{4} \div \frac{1}{2}$?

Why does long division work? What must students know as background before learning long division?

Section 1.1 Place value and Models for Arithmetic

Numbers are abstract ideas: 3 apples → 3 pears

Small numbers innate (say: built into our brains, chimpanzees recognize "3")

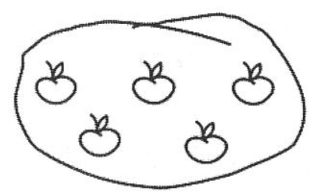
Def The whole numbers are 0, 1, 2, 3, ...
when used to count: Cardinal; when used to order: Ordinal

Taught by

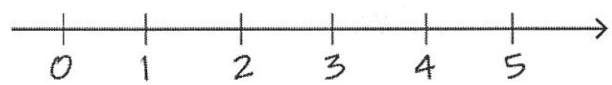
* Counting chants: "1, 2, 3, 4, ..."
* Counting exercises: "How many ______ ?"
* Patterns : : $\longleftrightarrow$ 4

based on

* Set model



* Measurement model



(say: number line – simplest meas. model)

Examples: Number of

(a) Weeks so far in the millennium
(b) People on Earth
(c) The height of the Sears Tower in feet
(d) Moons of Jupiter

Ask for: → Measurement / Set

(Say: This can be formalized with set theory, not used in elem. school)

We write numbers as symbols called <u>numerals</u>
(Say: A simple progression of 3 systems leads to the numeration system used today.)

I. Tally — I, II, III, IIII, ... intuitive, but try 989 !

II. Egyptian — Tallies up to 9, then

heelbone	ʌ	for 10
scroll	e	for 100
lotus	𓆼	for 1000

Ask: What does eeʌʌʌIIII represent? shorter, clear, but try 989 !

III. Decimal Numerals: uses ten symbols 0, 1, 2, ... 9 :

$$\begin{array}{ccc} e\,e & \wedge\wedge\wedge & |\,|\,|\,| \\ \downarrow & \downarrow & \downarrow \\ 2 & 3 & 4 \\ \text{hund.} & \text{tens} & \text{ones} \end{array}$$

the value of the digit depends "on its position within the number," this is called <u>place value</u>.

Advantages of the
<u>Decimal numeral system</u>:

* Easy to record very large #'s :

$$\underbrace{127}_{\text{trillions}}\,,\,\underbrace{671}_{\text{billions}}\,,\,\underbrace{238}_{\text{millions}}\,,\,\underbrace{541}_{\text{thousands}}\,,\,265$$

* Extends to record numbers with arbitrary accuracy:
 127.381

* <u>Much</u> easier to multiply and divide

* Used throughout the world.

<u>If time:</u>

Roman numerals are used & should be taught

Basic
- I
- X ⟷ 10
- C ⟷ 100 ("century")
- M ⟷ 1000 ("millennium")

Shortened by
- V ⟷ 5
- L ⟷ 50
- D ⟷ 500

MCCLXVII = <u>1267</u> ASK

784 = <u>DCCLXXX</u>IIII ASK

or

IV

"subtractive principle"

$\boxed{\text{HW}}$ Read introduction pg 1 – 5

Read S 1.1 Do HW set 1

(pg 6 – 10)

What to bring to class:
Ask students to bring PM 4A and 5A.

1.2 Place Value

last time: * whole #s
* models: set, measurement
* numerals: tally, Egyptian, Roman, Decimal
* place value: value of a digit is specified by its position within number

ex 3437
→ place value; 10, value 30
→ place value; 1000, value 3000

1.2 - Place value

Decimal System:
→ advantages: Place value (simpler notation)
→ disadvantages: Place value
(tricky concept! Students have difficulty all through elementary school!)

Say: Place value is so ingrained in adult minds. Difficult to appreciate importance & how hard it is to learn.

Decimal numbers formed by:

Place value Process
- Step 1 Form bundles of 1, 10, 100...
- Step 2 If necessary rebundle to ensure at most 9 bundles of each denomination
  think of : 10 pennies = 1 dime
- Step 3 Record number of each type of bundle in the appropriate position.

Say: Given a pile of pennies, dimes and dollars, how do you represent a 3-digit number? the act of creating a 3-digit numeral is a process! this process underlies nearly everything in elem. math.

Step 1 Put pennies in piles of ten

Step 2 Exchange each pile of 10 pennies for a dime.

Step 3 Exchange groups of 10 dimes for a dollar.

Example: K–3 problems which teach place value

* Counting by tens (step 1)
* Switching decades (what comes after 39? 59? 99? (step 2)
* thinking of 1482 as 14 hundreds + 82 ones or 1 thousand + 48 tens + 2 ones (step 2)
* What is 20 more than 247? (step 3)

Say: easier than "is more." It is place value not addition.

Models/Teaching Sequence:

1. Base 10 blocks

   SHOW CLASS – convey the idea of bundles of 10, but not the main idea of Place value (position determines value.)

2. Chip Model

hundreds	tens	ones
○○ ○○		○○ ○○ ○
4	0	5

← denominations

↑ place holder!

3. Expanded form:

3874 means 3 thousands + 8 hundreds + 7 tens + 4 ones

$$3000 + 800 + 70 + 4$$

Note: Egyptian numerals already in expanded form:

 = 231

200 30 1

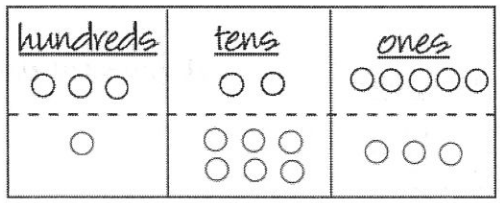
4. Decimal numerals: instead of 200 + 30 + 1 we write 231.

Place value in:

Adding – simple principle : separately add ones, tens, hundreds.

Example: (easier) 325 + 163

① Chip Model



② Expanded form:

$$\begin{array}{r} 300 + 20 + 5 \\ 100 + 60 + 3 \\ \hline 400 + 80 + 8 \end{array}$$

③ Decimals:

$$\begin{array}{r} 325 \\ +163 \\ \hline 488 \end{array}$$

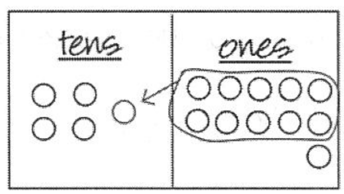
(note – each step further from actual counting)

* avoiding hard step ② – regrouping .
* can add columns in any order! Why?

Harder examples involve step ② – regrouping

composing: ("carrying" may be misleading, really exchanging)

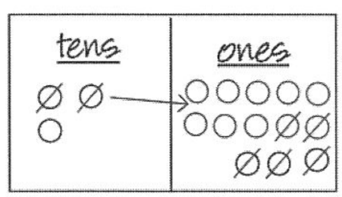
$$\begin{array}{r} 38 \\ +13 \\ \hline \end{array}$$



5 1

decompose: ("borrowing" misleading, exchanging)

$$\begin{array}{r} 33 \\ -15 \\ \hline \end{array}$$



1 8

(these problems are harder (with step ②) should be done later)

Note: "tens combinations" (1 + 9, 2 + 8, 3 + 7, 4 + 6, 5 + 5)
helps with regrouping

Ex: * 60 − 8 = 50 + (10 − 8) = 52
* 75 + 7 = 75 + 5 + 2 = 82

Place value in multiplication:

* "Multiply by 10" is done by "appending a zero"

$$\left.\begin{array}{r}\text{Replace each penny w/ dime}\\ \text{dime w/ dollar}\end{array}\right\} \Rightarrow \text{shift digits}$$

* Special feature of place value. Wouldn't work for 9!

1. Recall HW set 1 #7
e∩∩||| x 10 = ꝣee∩∩∩

2. pennies ⟶ dimes
dimes ⟶ dollars

Classroom Exercises: Show how Place Value is used to get answer

* 13 x 10 = 13 tens = 130
* 321 x 10 = (3 hundreds + 2 tens + 1) x 10
= (3 thousands + 2 hundreds + 1 ten) = 3210
* 5 x 50 = (5 x 5) x 10 = 25 tens = 250
* 24 x 100 = 2400

Ordering: (provides exercises which test & challenge place value understanding)

Fill in $<$ or $>$

* $57 \underline{>} 39$ good
* $64 \underline{>} 46$ good
* $57 \underline{<} 89$ not good! could get right answer for the wrong reason

Summary:

Easier problems – steps 1 & 3 only (teach 1st)
Harder – all 3 steps

HW – Read 1.2 & do HW set 2.

What to bring to class:
Ask students to bring PM 4A and 5A.

1.3 Addition

(go over #6 of HW 2 - shows them how students will struggle)

*Review - what is place value? (Good Exam question)
(value of a digit is specified by its position within number)

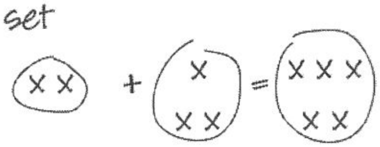
Addition:

2 + 3 = 5

addends or summands — sum

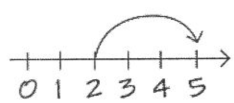
Explained with models:

* set

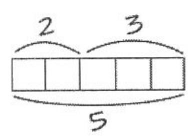


* measurement

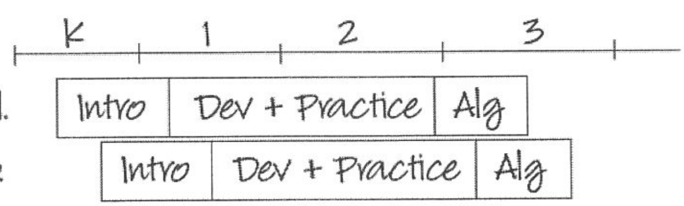
steps on number line



lengths of sticks



Teaching Stages:



Meaning of addition (counting on)

Mix of
- mental math
- worksheet
- short word problems

In chapter 3

say: Must balance. Don't want to hold math hostage by reading & writing skills!

Properties of Addition:

1 Additive Identity: "Adding zero does nothing"

Reason – set: Bag 1 has 7 chips, bag 2 has none. Pour contents of bags 1 and 2 into a 3rd bag. You get 7 in a new bag.

say: not really an addition fact – is really def. of zero

2 Any-order property: A list of whole numbers can be added in any order (with same answer)

Ex: $3 + 7 + 2 = (3 + 7) + 2 = (7 + 2) + 3 \ldots$

parentheses indicate which is done 1st

Reason – set: All chips thrown in same bag – order doesn't matter

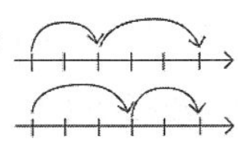
measurement: all lengths joined end to end – order doesn't matter

Special Cases:

a) For 2 numbers only Commutative Property – 2 numbers can be added in either order to yield same result.

Ex: $3 + 2 = 2 + 3$ Set: ○○○●●

●●●○○

Meas: 

b) For changing order of addition, but not order of numbers – called associative property

Ex: $(3 + 2) + 4 = 3 + (2 + 4)$

Together (a) + (b) give any-order prop.

Ex: $(2 + 3) + 4 = 3 + (2 + 4)$

true because

$$\underbrace{(2 + 3) + 4 = (3 + 2)}_{\text{comm.}} \underbrace{+ 4 = 3 + (2 + 4)}_{\text{assoc.}}$$

"Addition with in 20" = sums 0 + 0 to 10 + 10

-taught/learned using "thinking strategies"

Thinking/Teaching Strategies (in order to be taught)

1 Adding + 1, + 2 — Ex. $7 + 2 = 9$ easy by counting

2 Adding: 0 — $5 + 0 = 5$ natural, once taught

3 Commutativity : Pick the easier order — $\underbrace{2+7}_{\text{hard}} = \underbrace{7+2}_{\text{easy by } ①}$

4 Doubles: 3 + 3, 4 + 4, 5 + 5....

Ex: $5 + 5 \longrightarrow$ fingers

$6 + 6 \longrightarrow$ egg carton

5 Adding 10 — $6 + 10 = 16$ — Ask: What type of fact: Place Value

6 10's combinations — $\underbrace{9 + 1, 8 + 2}_{①}, 7 + 3, 6 + 4, \underbrace{5 + 5}_{④}$

7 Relating to doubles : "Mental Math"

$6 + 7 = (6 + 6) + 1$

$7 + 8 = (8 + 8) - 1$

8 Compensation : "Mental Math"

$9 + 6 = 10 + 5$

+1

"6 gives 1 to the 9"

Say: We will be doing lots of mental math to improve. Will use large numbers because smaller ones are already memorized.

Examples:

1. $38 + 32 = 40 + 30 = 70$ or $30 + 30 + (8 + 2)$

   +2 compensation — Place Value

2. $71 + 29$

   +1

3. $\underset{+4}{232 + 96} = 228 + 100 = 328$

4. $36 + 35 = (35 + 35) + 1 = 71$

doubles

5. $\underset{+7}{793 + 428} = 800 + 421 = 1221$

Summarize!

$\boxed{\text{HW}}$ Read Sect 1.3. Do HW Set 3.

What to bring to class:
Ask students to bring PM 4A and 5A.

1.4 Subtraction

*Review – Place Value
–Any Order Properties (Comm, Assoc)

Subtraction: Definition:

$13-5=$ ______ $\xleftrightarrow[\text{def}]{\text{by}}$ $5+$ ______ $=13$

"missing addend"

<u>Terminology:</u>

$13 - 5 = 8$

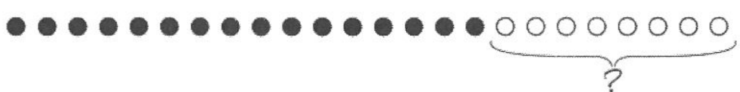
minuend subtrahend difference

Say: Need to know these terms to read teacher guides

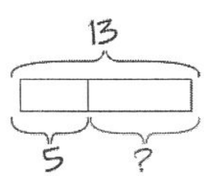
<u>3 interpertations:</u>

① <u>Part-whole</u> interpretation: "How much is the missing part?"

<u>Ex:</u> <u>Set Model:</u> there were 24 cars and trucks. 16 were cars.
How many were trucks?



<u>Meas. Model:</u>



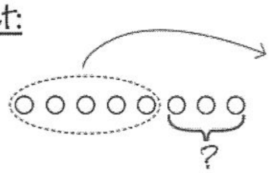
$13-5=$ ______

Say: Same as missing addend?

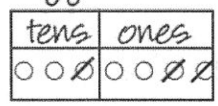
② **Take-away** interpretation: "How much is the remaining part?"

Ex: there were 8 pencils on the desk, 5 were picked up.
How many were left?

Set:

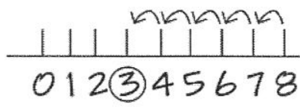


Note: With bigger numbers chip models help

Ex: 

$34 - 12$

Measurement:

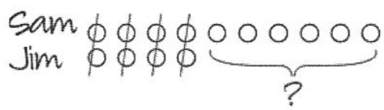


$8 - 5 = 3$

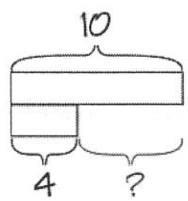
③ **Comparison** Interpretation: "How much more or less does one group have?"

Ex: Sam had 10 pencils. Jim had 4.
How many more pencils did Sam have?

Set:



Measurement:



Thinking/Learning Strategies for subtraction within 20.

a) Four-fact families

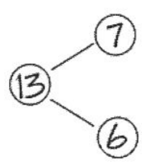
$6 + 7 = 13$

$7 + 6 = 13$ *aid to connecting + and −.

$13 - 7 = 6$

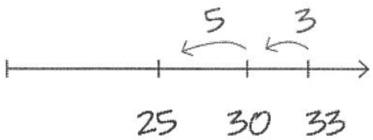
$13 - 6 = 7$ *reduce need for memorization

b) number bonds – display all four facts in one picture



c) <u>counting down:</u>

Ex: $33 - 8 = (33 - 3) - 5$
$= 30 - 5$
$= 25$

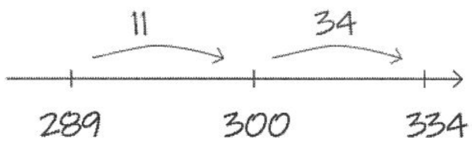


*use round numbers as stepping stones.

d) <u>counting up:</u>

Ex: $334 - 289$

start at 289. How far to 334?



$= 45$

Practice:

$132 - 94 =$

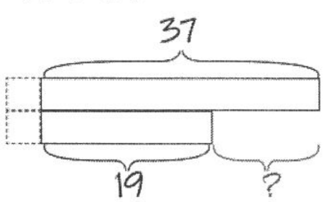
$1040 - 792 =$

<u>Mental Math:</u>

Recall <u>Compensation</u> for addition:

1. $67 + 59 = 66 + 60 = 126$
   $+1$
2. $769 + 51 = 770 + 50 = 820$
   $+1$
   place value w/rebundling
3. $37 - 19 = 38 - 20$ (<u>not</u> $36 - 20$)
   $= 18$
   place value

Let students try this!
Likely to make mistake.



4. $62 - 38 \longrightarrow 62 - 38 = 60 - 36$

$(-2)\ (-2)$

which is easier?

Make subtrahend nice!

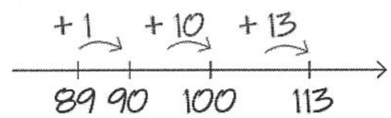
$62 - 38 = 64 - 40$

$(+2)\ (+2)$

<u>Compensation for $a - b$</u>: increse/decrease a&b by the same amound, usually by making b "nice" (ie – a multiple of 10)

counting up

5. $113 - 89 = (m1) = 1 + 10 + 13 = 24$



place value

$(m2) = 114 - 90 = 24$

compensation

6. $188 - 53 = 135$ place value w/ no rebundling (subtract tens, ones)

7. $1859 - 532 = 1327$ just place value

$\boxed{\text{HW}}$ Read 1.4, do HW # 4.

What to bring to class:
Ask students to bring PM 4A and 5A.

1.5 – Multiplication

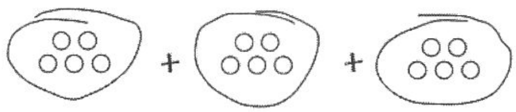
Say: What is multiplication? We know $\underset{\text{factors}}{3 \times 6} = \overset{\text{product}}{18}$. What does this mean?

<u>Def</u>: Multiplication of whole numbers is repeated addition.

$3 \times 5 = 3$ groups of 5 or $5 + 5 + 5$

<u>Models</u>:

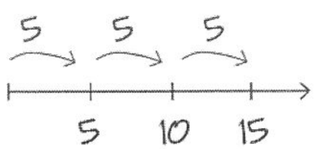
a) <u>set Model</u>:



"3 groups of 5 ones"

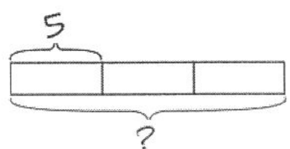
b) <u>Measurement</u>:

number line



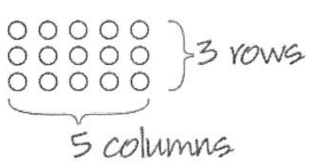
"3 steps of size 5"

bar diagram



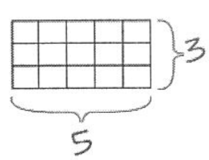
c) <u>Rectangular array</u>:

<u>set</u>



or

<u>Meas</u>:



"area model"

Multiplication properties: (just through examples at this stage)

① Multiplication Identity (multiplication by 1)

what is 5 groups of 1? $5 \times 1 = 1 + 1 + 1 + 1 + 1 = 5$

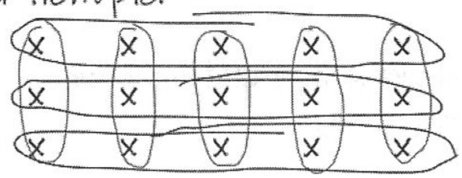
what is 1 group of 5? $1 \times 5 = 5$

② Commutative Property: $3 \times 5 = 5 \times 3$

Say: not obvious that 3 groups of 5 = 5 groups of 3

$5 + 5 + 5 = 3 + 3 + 3 + 3 + 3$

Clear from pic:

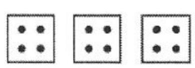


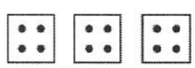
Rows = 3 groups of 5

columns = 5 groups of 3

*not obvious from other models.

③ Associative Property: $2 \times (3 \times 4) = (2 \times 3) \times 4$

 2 rows of (3 x 4) dots

 = (2 x 3) boxes of 4 dots

② & ③ together give the Any - order Property: A list of whole numbers can be multiplied in any order.

Ex: $3 \times (4 \times 2) = 2 \times (3 \times 4) = (3 \times 2) \times 4$ etc.

Say: remember parentheses show which to mult. first.

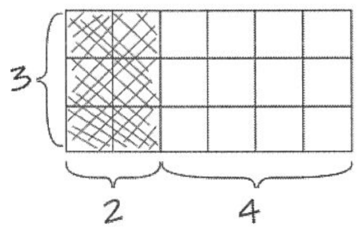
Last property involves multiplication & addition.

④ <u>Distributive property:</u>

shaded

$3 \times (2 + 4) = (3 \times 2) + (3 \times 4)$

unshaded



used with place value:

$5 \times 13 = 5 \times (10 + 3) = 50 + 15 = 65$

" 13 fives = 10 fives + 3 fives"

Say: any - order & Distributive Properties are involved in <u>most</u> arithmetic calculations.

<u>Teaching/Thinking Strategies</u> (3 teaching phases)

A. Intro phase– (endof grade 1)

- a) x 2 doubles (know from add)
- b) x3 (taught)
- c) x 0, x 1 (natural)
- d) x 10 (place value)

B. Mental Math & Word Problems (2nd grade)

- a) x 5 –skip counting, mental math
- b) commutative prop – rect array
- c) x 9 – think $6 \times 9 = 6 \times 10 - 6$ (mental math)
- d) 3×40, 20×30 – place value
- e) Practice using Any-order & Distributative props. – Much practice, models

C. Close topic (by end of 3rd grade)

a) squares 3 x 3, 4 x 4, . . . , 9 x 9 – learned

b) Remaining facts – memorized.

ex. 8 x 7

Say: knowing multiplication facts necessary for fluency. Short term memory freed up.

<u>Mental Math:</u>

① x 4 double twice $17 \times 4 = 34 \times 2 = 68$

② x 8 double 3 times $16 \times 8 = 32 \times 4 = 64 \times 2 = 128$

③ x 5 (1/2 number) x 10
or x 10 then half

Practice:

5 x 18
5 x 42
242 x 5
1282 x 5
15 x 5
43 x 5
165 x 5

④ x 9

$9 \times 7 =$ think $10 - 1$ $7(10 - 1) = 70 - 7 = 63$

$9 \times 13 = 13(10 - 1) = 130 - 13 = 117$

$9 \times 24 =$

$9 \times 130 =$

HW Read 1.5
HW #5
bring PM 3A to next class

What to bring to class:
Ask students to bring PM 4A and 5A.

1.6 – Division

<u>Def:</u> Division is related to multiplication by "missing factor"
(say: similar to missing addend with subtraction)

Ex:

$24 \div 4 = \_$ ⟵means⟶ $4 \times \_ = 24$

dividend divisor quotient   factors product

<u>Note:</u> * Write $12 \div 4$ not $12/4$ until fractions are mastered.

*No new facts needed- relate to multiplication

<u>4-fact families:</u>

$5 \times 7 = 35$

$7 \times 5 = 35$ *reduces need for memorization

$35 \div 7 = 5$

$35 \div 5 = 7$

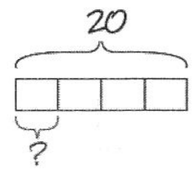
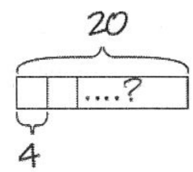
Division is about making groups. Illustrate using

Set Model
Measurement Model

ask

preferred, since set model is inefficient for large numbers

2 interpretations: Ex: $20 \div 4$ means

Interpretation	Int. question	Diagram
Partitive	"20 is 4 groups of what?"	
Measurement	"20 is how many 4s?"	

say: don't confuse with meas. model

Say: these are important & very different.

Partitive = how big is each part?

Measurement = think of measuring w/ ruler length 4. How many parts?

*teacher knowledge – know how to vary word problems when teaching.

Examples: Primary 3A (have students open book)

- pg 45 #2
- pg 47 #9
- pg 48 #9
- pg 65 #6–10

Ask:

1) Partitive or Measurement?

easier { 2) Interpretive question?
3) Which diagram type? }

Mental Math:

Divide by 4: (divide by 2 twice)

$84 \div 4$	$84 \xrightarrow{\div 2} 42 \xrightarrow{\div 2} 21$
$128 \div 4$	$\underline{\underline{12}}8 \xrightarrow{\div 2} 64 \xrightarrow{\div 2} 32$
$1264 \div 4$	$\underline{\underline{12}}64 \longrightarrow 6\underline{\underline{32}} \longrightarrow 316$
$4164 \div 4$	$4\underline{\underline{16}}4 \longrightarrow 2082 \longrightarrow 1041$

*explain how underlined digits are used.

Divide by 5: (divide by 10 and double) or (double, then ÷ 10):

$330 \div 5$

$1280 \div 5$

$135 \div 5$

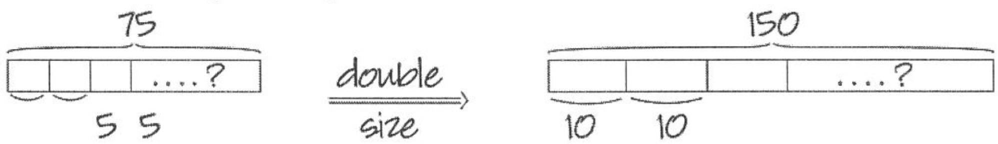
$75 \div 5$

$415 \div 5$

Compensation:

$75 \div 5 = 150 \div 10$

doubling both gives same answer



75 is how many 5's? 150 is how many 10's?

$140 \div 5 = 280 \div 10 = 28$

$135 \div 15 = 270 \div 30 = 27 \div 3 = 9$

compensation again!

$1400 \div 35 = 280\cancel{0} \div 7\cancel{0} = 40$

$150 \div 6 = 50 \div 2 = 25$

$(\div 3)$ $(\div 3)$

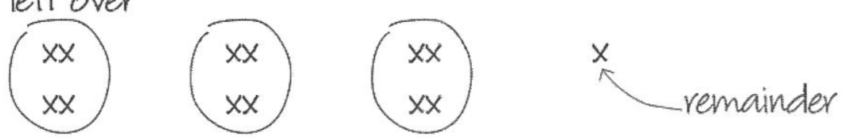
Division by 0 is undefined

Case 1: $8 \div 0$	8 is how many 0's?	No answer
Case 2: $0 \div 0$	0 is how many 0's?	Too many answers!

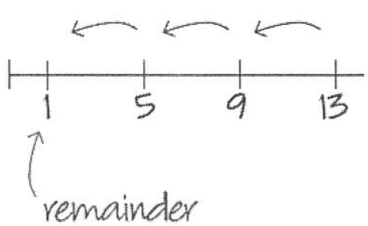
*so _÷0 is undefined because either way it doesn't specify a number!

Remainders: Amount left over

set: $13 \div 4$



Measurement:



Quotient Remainder Theorem:

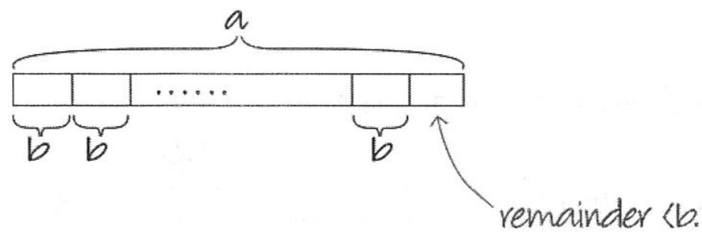
For any whole numbers a&b ($b \neq 0$) there are unique whole numbers.

q (quotient) and r (remainder) so that

$$a = bq + r \qquad 0 \leq r < b$$

Explanation: a=(q groups of b) + remainder

*measure bar of length a with ruler of length b:



$\boxed{\text{HW}}$ Read 1.6 & 1.7

Do HW 6

Bring 3A & 5A

*if time - bar diagrams for 3cd HW6, 4ab, 6

*HW 5 - go over 4,6,7

What to bring to class:
Ask students to bring PM 4A and 5A.

2.1 – 2.3 Mental Math and Word Problems

(2 days, all of chp. 3)

Mental Math

* Using distributive property

$108 \times 6 = (100 + 8) \times 6 = 600 + 48 = 648$

$165 \div 15 = (150 + 15) \div 15 = 10 + 1 = 11$

$410 \div 13 =$ (think 30 13's makes 390,
+1 13 makes 403
7 remain)
$= 31$ R 7

* Compensation:

for +: $\overset{+13}{87 + 56} = 100 + 43 = 143$

for –: $87 - 56 = 81 - 50 = 31$

for x: $25 \times 36 = (25 \times 4) \times 9 = 900$

for ÷: $204 \div 6 = 102 \div 3 =$
$(90 + 12) \div 3 = 30 + 4$

Word Problems

Should be
* Short, Clear, Succinct
* Interesting but not Flowery
* Realistic but not contrived
* Self contained and well defined

(Say: there may be many ways to answer, but only one, or at most, several specific answers)

In sets which
* are not varied in context (different models, etc.), not underlying math
* build up 1 step → 2 step → multi step

We will do many word problems:

Examples:

Note to instructor: Do selection from Sing 3A pgs. 66-67
* Do some at top of page by mental Math.
* Do some word problems (w/Teacher's solutions)
  For division prob., ask for interpretation

Teacher's Solutions (Graded on this criteria)

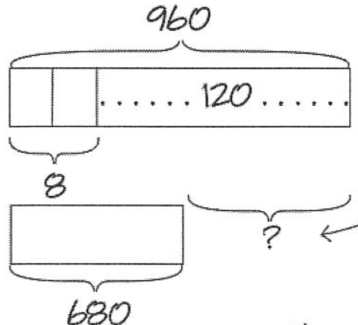
Example

There are 8 stamps in a set. Gopal bought 120 sets. After selling some stamps, he had 680 left. How many did he sell?

Teacher's Solution

unkown clearly labeled

Diagram completely describes problem.



1 unit = 8

120 units = 960

960 − 680 = 280

Logic explained

He sold 280 stamps.

answer clearly stated

Have students do problems 7 – 9 on pg 90 sing 3A time permitting. Have students present Teacher's solutions (T.S.).

$\boxed{\text{HW}}$ Do HW set   Read section 22 & section 23, then do HW set 7.
Bring sing 3A & 5A to next class

(*) Note to Instructor:

* Put students in pairs of 2, divide class into 3 sets.
* Assign 3 problems at a time, one to each $\frac{1}{3}$ of the class.
* Give only 2 – 3 minutes – strictly timed.
* While students work, write questions on the Blackboard.
* Select 3 pairs of students to go up and present Teachers solution.

Do Sing 3A; pg 54 prob 10 – 12
pg 55 prob 9 – 11
$\boxed{\text{pg 56} \quad \text{prob 9 – 11}}$ if time

<u>Multi-step word</u> problems combine 2 different operations – the most interesting cannot be classified as a +, –, x, ÷ problem.

Do Sing 5 as in (*), pg

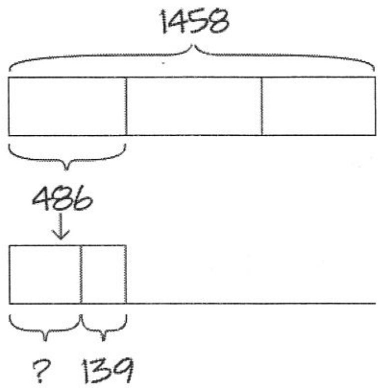
Book 5A, pg 22 – 23

* Go over pg 22
* Work thru #1 – 3 with class
* groups of 2 for # 1 – 4 of practice 1D of Sing 5A

then present:

* 5A pg63 #31

Peter, John, and Dan shared $1458 equally. Peter used part of his share to buy a bicycle and had $139 left. What was the cost of the bicycle?



3 units = 1458

1 unit = 1458 ÷ 3 = 486

$$\begin{array}{r} 486 \\ 3\overline{)1458} \\ \underline{12} \\ 25 \\ \underline{24} \\ 18 \end{array}$$

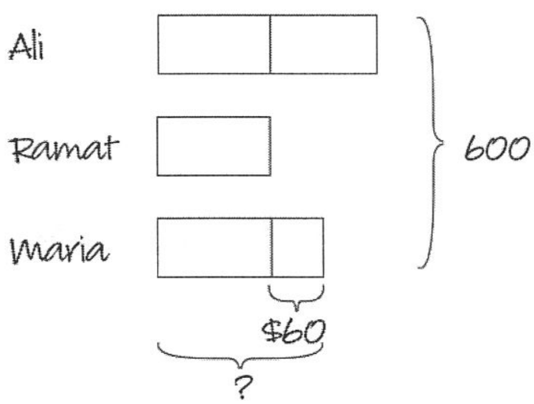
486 − 139 = 487 − 140

= $347

The bike cost $347.

* 5A pg90 #16

Ali saved twice as much as Ramat. Maria saved $60 more than Ramat. If they saved $600 altogether, how much did Maria save?



4 units + $60 = $600

4 units = $540

1 unit = $135

135 + 60 = 195

Maria saved $195.

In groups of 2.
Assign #2, #4, #8 on page 25 of Sing 5A

Send to the board after 4 min to present Teacher's solutions

If time, give a quiz.

$\boxed{\text{HW}}$ Do HW set 6 and HW set 8

(Say: we did some of these problems)

What to bring to class:
Ask students to bring PM 4A and 5A.

3.1 & 3.2 – Add/Subtraction Algorithm

3.1 – Addition Algorithm:

Def: An algorithm is a systematic, step-by-step procedure to solve a class of problems.

Ex: spell check, Addition Algorithm

Algorithms taught because:

* Always work – builds confidence
* become automatic – frees up memory
* completes topic & establishes "level playing field"

say: everyone in the class can +, −, x, ÷

Prerequisites to addition algorithm:

1) count to 1000
2) 1-digit add facts
3) 2-digit Mental math
4) Expanded form via chip models
   a) Add w/ in same denomination
   b) Rebundling "10 dimes = 1 dollar"

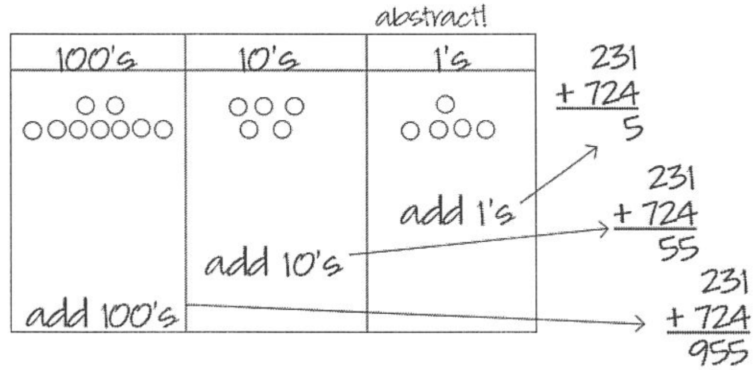
Teaching Stages:

1) No rebundling – simple idea: add ones, tens, hundreds separately.
   (say: no step (ii) of place value process)

* be sure to do chip model & #'s at the same time! Make connection between model & abstract!

Ex:

Add 231 + 724



quickly move

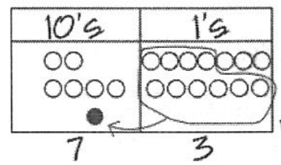
Concrete ⟶ Pictorial ⟶ Abstract

coins, etc — chip model — numbers only

2) Rebundling

a) 10 ones = 1 tens

EX:
$$\begin{array}{r} {}^{1} \\ 27 \\ +\ 46 \\ \hline 73 \end{array}$$



Call this "rebundling a ten"

b) 10 tens = 1 hundred

EX:
$$\begin{array}{r} 283 \\ +\ 142 \\ \hline \end{array}$$

c) combinations

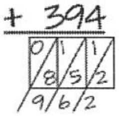
EX:
$$\begin{array}{r} 295 \\ +\ 326 \\ \hline \end{array}$$

Have students try:
$$\begin{array}{r} 7065 \\ +3836 \\ \hline \end{array}$$

3) <u>Alternative Algorithm</u>: LATTICE METHOD

note: works <u>only</u> when place-value process known before hand otherwise memorization w/o understanding

<u>EX</u>:
$$\begin{array}{r} 568 \\ +\ 394 \\ \hline \end{array}$$



<u>Try</u>:
$$\begin{array}{r} 3576 \\ +\ 4829 \\ \hline \end{array}$$

<u>say</u>: ① Chip models used <u>briefly</u> to introduce algorithm. then only numbers

② Don't draw chip models on HW unless specifically asked.

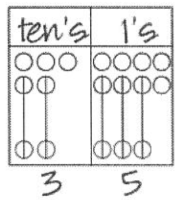
3.2 Subtraction Algorithm: (done similar to addition)

Teaching Stages:

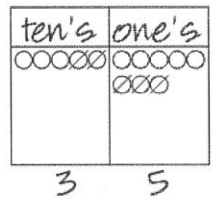
1) No rebundling (subtract ones, tens, etc)

Ex: $\begin{array}{r} 58 \\ \underline{-\ 23} \\ 35 \end{array}$

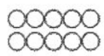
Comparison:



Take-away:

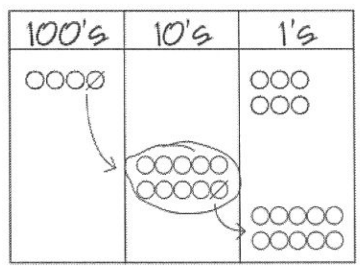


2) Rebundling: (decomposing a ten, splitting a bundle, etc)

10 $\longrightarrow$ 

Hardest case:

Ex: $\begin{array}{r} 3\ 9\ 16 \\ \cancel{4}\ \cancel{0}\ \cancel{6} \\ \underline{-\ 1\ 3\ 9} \end{array}$



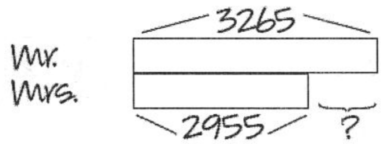
Teaching Remark: Don't let student invent their own algorithms

1) Mental Math is for creativity
2) Hard on teacher
3) Algorithm is the structured conclusion.

Practice algorithms with word problems

<u>Ex</u>: Mr. Smith earned \$3,265. His wife earned \$2,955.
How much more money did he earn than his wife?
(individual – 2 minutes)

Teacher's Solution



$$\begin{array}{r} {}^{2}\,{}^{12} \\ \cancel{3}\,\cancel{2}\,6\,5 \\ -\,2\,9\,5\,5 \\ \hline 3\,1\,0 \end{array}$$

He earned \$310 more.

<u>Alternate Algorithm</u>: "Subtract from 10"

Ex:
$$\begin{array}{r} {}^{2}\,{}^{10} \\ \cancel{3}\,7 \\ -\,1\,9 \\ \hline 1\,8 \end{array} \longleftarrow \text{think: } \underline{(10-9)} + 7$$

ten's complement

<u>Try</u>:
$$\begin{array}{r} 61 \\ -\,37 \\ \hline \end{array} \qquad \begin{array}{r} 272 \\ -\,138 \\ \hline \end{array}$$

<u>Adv</u>: only need 10's complements
<u>Dis</u>: not standard, must add too!

<u>Common Error</u>:

$$\begin{array}{r} 45 \\ -\,7 \\ \hline 42 \end{array}$$

What's the mistake?
① "7 – 5" instead "15 – 7"
② "7+5" w/no rebundling

$\boxed{\text{HW}}$ Read 3.1 & 3.2
HW 10 & 11

What to bring to class:
Ask students to bring PM 4A and 5A.

3.3-Multiplication Algorithm

Mental Math: (in-class ex 1-min each step)

Step 1: $11^2=121$
$12^2=144$
$13^2=169$
$14^2=196$
$15^2=225$
$16^2=256$

Step2: Use facts to calc
a) 11x12
b) 14x15
c) 16x18
d) 22x12

Teacher knowledge: Using Place Value (PV) & Distributive Property (DP) every "x" can be re duced to a series of 1-digit "x".

Stage 1: 1-digit x (2 or 3 digit)

Ex: 3x145	= 3x (100+40+5)	PV (Expanded Form)
	= 300+120+15	DP
	=435	PV

Stage 2: 2-digit x (2 or 3 digit)

Ex: 23x145	= (20+3) x 145	PV (Expanded Form)
	= (20x145) + (3x145)	DP
	= 10 (2x145) + (3x145)	Any-order

stage 1

*Stage 2 problems reduce to stage 1 which reduce to 1-digit "x".

Teaching Remarks: PV & DP should be repeatedly covered before & during the teaching of the algorithm.

Models: Distributive Property: rect. array
Place Value: chip model

Teaching Stages:

1) (Grade 3&4)

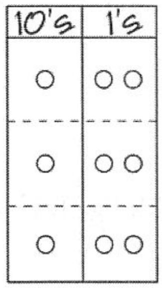
a) 1-digit mult. in different P.V.

6	60 ← 6 tens	600	(PM 3A p49)
x4	x 4	x 4	
24	240 ← 24 tens	2400	

b) Multiplication without regrouping. (PM 3A p50 pr 2)

Ex: 3x12

1) chip diagram



3 6

2) Algorithm Format

$$\begin{array}{r} 12 \\ \underline{\times\ 3} \\ 36 \end{array}$$

3) step-by-step

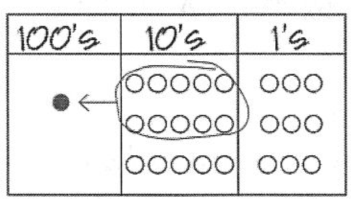
$$\begin{array}{r} 12 \\ \underline{\times\ 3} \\ 6 \\ \underline{+\ 30} \\ 36 \end{array}$$

4) Distributive Property highlighted

$$\begin{array}{r} (10+2) \\ \underline{\times\ \ \ 3} \\ 30+6 \end{array}$$

c) Mult w/ regrouping in top denomination

$$\begin{array}{r} 53 \\ \underline{\times 3} \\ 159 \end{array}$$



* do model & abstract at the same time

*why easier in top den?

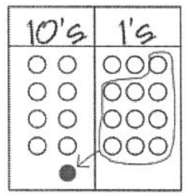
*Mult ones: 3x3=9

*Then (5x3) tens = 15 tens

*Then regroup

d) Mult with regrouping in lower denominations

Ex:
```
 1
 23
 x4
 92
```



*3x4=12 ones

=1 ten & 2 ones

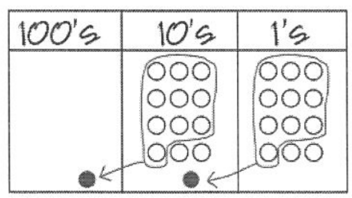
* (2x4) tens = 8 tens

+ 1 more

9 tens

e) Double regrouping

Ex:
```
 1
 33
 x4
 132
```



have students try
in gps of 2
(use chip model)

*4x3=12 ones

=1 ten + 2 ones

*4x3 tens + 1 ten = 12 tens + 1 ten

=1 hun + 3 tens

Stage 2: (Grades 4 & 5)

a) Review stage 1

b) Mult by 10, 20, . . . , 90

Ex: 12 x 40 = 12 x 4 tens = 48 tens = 480

or
```
  12
 x40
 480
```

stage 1
& shift Place Value

c) Together

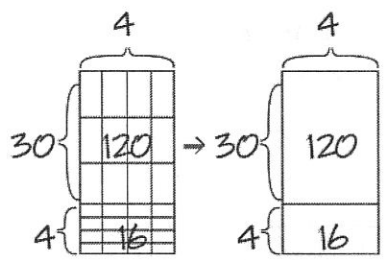
$$\begin{array}{r}24\\ \times\ 13\\ \hline 72\\ {}^{1}240\\ \hline 312\end{array}\quad\begin{array}{l}\\ \\ \longleftarrow 24\times 3\\ \longleftarrow 24\times 10\\ \longleftarrow \text{total}\end{array}$$

* then practice in Word Problems

<u>Alternate Algorithm:</u> (say: still uses place value & distributive property)

<u>Ex 1:</u>

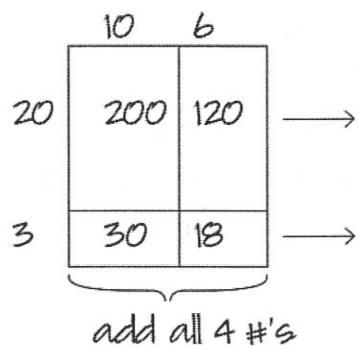
34 x 4



$$\begin{array}{r}34\\ \times\ \ 4\\ \hline 16\\ +\ 120\\ \hline 136\end{array}$$

<u>Ex 2:</u>

$$\begin{array}{r}23\\ \times\ 16\\ \hline 18\\ 120\\ 30\\ 200\\ \hline 368\end{array}$$

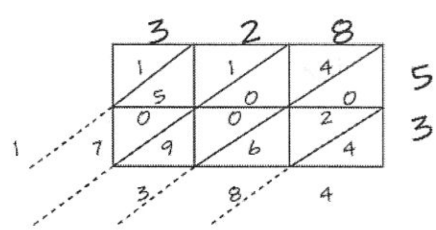


shorten

$$\begin{array}{r}320\\ 48\\ \hline 368\end{array}$$

<u>Lattice Method:</u> *Be very careful of arrangement.

$$\begin{array}{r}328\\ \times\ \ 53\\ \hline 17{,}384\end{array}$$



Uses:
*1-dig mul
*Place value
*lattice=array

Try: 2874
x 19

$\boxed{\text{HW}}$ Read 3.3. HW 12 Memorize 11^2–20^2

What to bring to class:
Ask students to bring PM 4A and 5A.

3.4 Long Division

* *Most Important algorithm taught in elementary school
* * Helps understand "successive approx"
  (comes up often in math, science, computer science)
* * Relating fractions & decimals; irrationals

Prereq's

1) Place value
2) 1-digit "x" & "÷"
3) quotient & remainder
4) Long div. notation $28 \div 3 \longleftrightarrow 3\overline{)28}$
5) Estimation (in 3.5)

Taught in 3 stages (can use Partitive or Measurement)

ask for these first.

1) 1-digit divisors $3\overline{)768}$
2) Estimation: Transition step
3) 2-digit divisors

$56\overline{)4832}$ $56\overline{)4832}$ ∘ ∘ ∘ think $\approx 60\overline{)4800}$ with quotient 30

uses estimation

Stage 1:

Partitive approach

Ex: Ms Davis divided 13 candies equally among 4 children.

Each child had_____candies
with a total of_____distributed
and____________left over
} 3 # to keep track of

[Note: this is quotient-remainder thm: $13 = \underbrace{(4\times\_)}_{\_} + \_$]

$13 \div 4 = 3R1$

Record as

$$\begin{array}{rl} 3 & \longleftarrow \text{ \# to each child} \\ 4\overline{)13} & \\ \underline{12} & \longleftarrow \text{ \# distributed} \\ 1 & \longleftarrow \text{ left to distribute} \end{array}$$

Ex: Ann, Beth & Camile split \$8.82 equally. How much did each get?

1) Distribute dollars

two left. convert to dimes
⟹ 28 dimes

$$\begin{array}{rl} 2 & \longleftarrow \text{ \$2 each} \\ 3\overline{)8{:}82} & \\ \underline{6} & \longleftarrow \text{ distribute \$6} \\ 2{:}8 & \end{array}$$

2) Distribute dimes

1 dime left. Convert to pennies
⟹ 12 pennies

$$\begin{array}{rl} 29 & \longleftarrow \text{ 9 dimes each} \\ 3\overline{)8{:}8{:}2} & \\ \underline{6} & \\ 2{:}8 & \\ \underline{2{:}7} & \longleftarrow \text{ 27 dimes distributed} \\ 1{:}2 & \end{array}$$

3) Distribute pennies

$$\begin{array}{rl} 294 & \longleftarrow \text{ 4 pennies each} \\ 3\overline{)882} & \\ \underline{6} & \\ 28 & \\ \underline{27} & \\ 12 & \\ \underline{12} & \longleftarrow \text{ 12 distributed} \\ 0 & \longleftarrow \text{ 0 left} \end{array}$$

Done

<u>Observe:</u>

- Essential role of Place value
- Each step same distribute, record, make change
- Steps give better & better approximations

  200 ⇝ 290 ⇝ 294

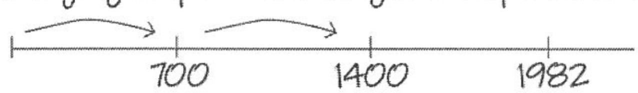
  (say: one more correct digit each time)

<u>Repeated subtraction approach</u> (say: similar–uses meas model)

<u>Ex</u>: Find 1984 ÷ 7

think: How many 7's in 1948? (what type of interpretation? Multi-Digit)

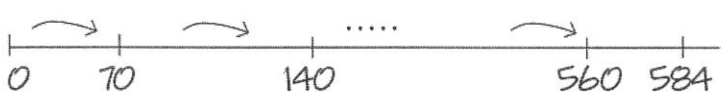
1) Flying leaps – 100 length 7 steps at a time



$$\begin{array}{r} 2 \\ 7\overline{)1984} \\ \underline{1400} \\ 584 \end{array}$$

200 steps = 1400
leaves 584 to go

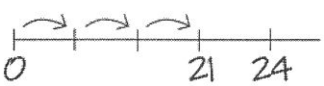
2) Giant Leaps – 10 length 7 steps at a time



$$\begin{array}{r} 283 \\ 7\overline{)1984} \\ \underline{1400} \\ 584 \\ \underline{560} \\ 24 \\ \underline{21} \\ 3 \end{array}$$

⟹ 80 steps = 560
leaves 24

3) Length 7 steps



3 steps = 21, leaves 3

this completes stage 1. Next time stage 2 & then 3.

$\boxed{\text{HW}}$ – Read 3.4

do HW #13

bring PM 4A & Workbook 5A

What to bring to class:
Ask students to bring PM 4A and 5A.

$\boxed{\text{3.5–Estimation}}$ – transition to multi-digit long division

Division with 1-digit divisions (stage 1) is easy because we know 1 through 10x10 mult. facts.

Ex:

$$\begin{array}{r} 62 \\ 6\overline{)372} \\ \underline{36} \\ 12 \\ \underline{12} \\ 0 \end{array}$$

(give students 30 sec.)

ask: What did you have to know? 6×6, 6×2

If we knew mult table for 16, then long division by 16 would be easy:

use Mental Math to fill out

$16 \times 1 = 16$
$16 \times 2 = 32$
$16 \times 3 = 48$
$16 \times 4 = 64$
$16 \times 5 = 80$
$16 \times 6 = 96$
$16 \times 7 = 112$
$16 \times 8 = 128$
$16 \times 9 = 144$

$$\begin{array}{r} 572 \\ 16\overline{)9152} \\ \underline{80} \\ 115 \\ \underline{112} \\ 32 \end{array}$$

Ex: find $16{,}500{,}188 \div 29$ (groups of 2, 2 min)

start & pt out

$29 \times 1 = 29$
$29 \times 2 = 58$
$\underline{29 \times 3 = 87}$
$29 \times 4 = 4 \times 30 - 4 = 116$
$\times 5 = 145$
$\times 6 = 174$
$\times 7 = 203$
$\times 8 = 232$
$\times 9 = 261$

$$\begin{array}{r} 568972 \\ 29\overline{)16500188} \\ \underline{145} \\ 200 \\ \underline{174} \\ 260 \\ \underline{232} \\ 281 \\ \underline{261} \\ 208 \\ \underline{203} \\ 58 \\ \underline{58} \\ 0 \end{array}$$

When we don't have such tables we can estimate instead.

Def: Estimation is the process of finding an approximate answer (the "estimate") to a given computation.

Used:

*when only approx answers are required.

"Roughly how many hours in 400 min of cellphone time?"

$400 \div 60 \approx 420 \div 60 = 7$ hrs

*to check answers to complex calc's

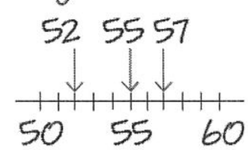
$$\frac{123.234 \times 1.8873}{3.256} \approx \frac{120 \times 2}{3} \approx 80$$

Say: can also estimate measurements (how much does this child weigh?) but we won't discuss that.

Estimation uses: Mental Math, Place Value, Round-off

Round up 5's algorithm: (easily taught using number line)

Ex: Round to nearest 10



$52 \rightsquigarrow 50$

$57 \rightsquigarrow 60$

$55 \rightsquigarrow 60$

(round mid pt up by convention)

Ex: Round 2735 to nearest
10 $\rightsquigarrow$ 2740
100 $\rightsquigarrow$ 2700
1000 $\rightsquigarrow$ 3000

Look at PM 4A pg 13 & 15

* It's an algorithm
* quickly developed, pg 13 – # line, pg 14 Arithmetic exer, pg 15, Early word prob.

Estimation Techniques

1. Round to Compatible #'s

   * $405 \times 243 \approx 400 \times 25 = 4 \times 100 \times 25 = 100 \times 100 = 10{,}000$
   * $4778 \div 62 \approx 4800 \div 60 = 80$

2. Front end – truncate after 1st or 2nd largest denominations

   * $476 + 531 \approx$ 47 tens + 53 tens = 100 tens
   * $356 + 622 \approx$ 3 hundreds + 6 hundreds = 900

3. Front end with Adjustments

   * $498 + 251 \xrightarrow{\text{front end}} 400 + 200 = 600$

     $\xrightarrow{\text{adjust}} 600 + (100 + 50) = 750$

     $\approx 98 + 51$

4. High – Low Range estimate: Get upper/lower estimate by consistently rounding up or down

   * Addition 587 + 734

500		583		600
+700		+734		+800
1200	<	actual	<	1400
low				high

   * Multiplication 386 x 892

300		386		400
x800		x892		x900
240000	<	actual	<	360000
low				high

Simple Estimation – Rounding to 1-digit arithmetic problems
(ex: 78 – 6, 7 x 8, 36 – 9)

Have students open PM 5A Workbook.

* Do Pg 11 in Workbooks (2-minutes)
  say: these are "1-digit" arithmetic problems.
  Goal is to reduce complicated problems to ones like these.
* Pg 12 do a, b, e, f , Notice rounding & 1-digit.
* Pg 13 do a, b, e, f , " " "

then combine with PV:

* Pg 14 do 1a, b PV
  c (not 1-digit so harder)
* Pg 15
  - 2a (on board)

$$326 \times 47 \approx 300 \times 50 = 15 \times 10 \times 10 \times 10 = 15000$$

rounding "same number of zeros"

  - do 2bc
  - do 3
    Instructor puts on board:

$$28 \times 229 \times 30 \times 200 = \$600$$

round up round down (compensation!!)

optional: Always an issue of how accurate to be:

$$16.1 \times 27.3 \approx \begin{cases} 16 \times 27 \\ 15 \times 30 \\ 10 \times 30 \end{cases}$$

which to pick? question for students & teachers.

Teachers/books should be clear about

* expected accuracy
* method
* use numbers with "obvious" estimate

If time: PM 5A WB Pg 16–17
call on students to answer
1a, 1b, 1c, 1d

Do 2a verbally (say: reduce to previous type of problem)
Have students do 2b, 3

put on board.

$$\underset{\substack{\uparrow \\ \text{round} \\ \text{down}}}{805} \div \underset{\substack{\uparrow \\ \text{round} \\ \text{up}}}{28} \approx 800 \div 30$$

(compensation !!)

$\boxed{\text{HW}}$ - Read 3.5
Do HW #14

What to bring to class:
Ask students to bring PM 4A and 5A.

3.6-Multidigit Long Division

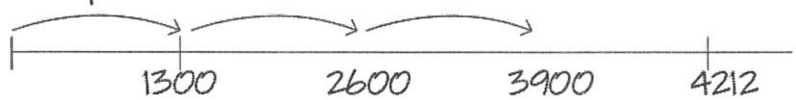
Ex 1:

$$\begin{array}{r} 324 \\ 13\overline{)4212} \\ \underline{39} \\ 31 \\ \underline{26} \\ 52 \\ \underline{52} \\ 0 \end{array}$$

have students do

① Interpretations: Partitive-distribute \$42 to 13 people...
⇒ \$3 each, \$39 total, \$3 left,...

Repeated Subtraction-



these interpretations still apply, but build from 1-digit division

② Facts needed: 13x3
13x2 ← no point making a table
13x4

EX 2:

$$\begin{array}{r} 82 \\ 19\overline{)1558} \\ \underline{-152} \\ 38 \\ \underline{38} \\ 0 \end{array}$$

How many 19's in 155?

Estimate:

$$\begin{array}{r} 8 \\ 20\overline{)160} \end{array}$$

Calculate: 19 x 8 = 160 − 8 = 152

Ex 3: $28\overline{)2076}$

How many 28's in 2076?
Let's relate to 1-digit fact by rounding!

$28\overline{)2076}$

round to nerest 10

round in same direction to hundreds

$\begin{array}{r} 7 \\ 30\overline{)2100} \end{array}$

$2100 \div 30 = 210 \div 3 = 70$

$$\begin{array}{r} 74 \text{ R}4 \\ 28\overline{)2076} \\ \underline{-196} \\ 116 \\ \underline{112} \\ 4 \end{array}$$

$$\begin{array}{r} {}^{5} \\ 28 \\ \underline{\times \ 7} \\ 196 \end{array}$$

total distributed

$$\begin{array}{r} 4 \\ 30\overline{)120} \end{array}$$

$$\begin{array}{r} {}^{3} \\ 28 \\ \underline{\times \ 4} \\ 112 \end{array}$$

After each distribution do:

Check: is $0 \leq$ remainder $<$ divisor?

yes- "make change" -ie- shift place value, bring down next digit, repeat.

no- Revise quotient by *try again, or

*add ± 1 copy of divisor

Ex 4:

$$\begin{array}{r} 7 \\ 26\overline{)2095} \\ \underline{-182} \\ 27 \end{array}$$

distribute

total distributed

$$\begin{array}{r} 7 \\ 30\overline{)2100} \end{array} \qquad \begin{array}{r} 7 \\ 3\overline{)210} \end{array}$$

Check: too large!

$$\begin{array}{r} 4 \\ 26 \\ \underline{\times\ 7} \\ 182 \end{array}$$

$$\begin{array}{r} 8 \\ \cancel{7}0 \\ 26\overline{)2095} \\ \underline{182} \\ 27 \\ \underline{26} \\ 15 \end{array}$$

common mistake – forgetting the "0".

check; okay!

Answer 80 R15

Ex 5: (Individual, 2 mins)

$$\begin{array}{r} 85 \\ 77\overline{)6568} \\ \underline{616} \\ 408 \\ \underline{385} \\ 23 \end{array}$$

check

$$\begin{array}{r} 8 \\ 80\overline{)6600} \end{array}$$

$$\begin{array}{r} 5 \\ 77 \\ \underline{\times 8} \\ 616 \end{array}$$

Ans: 85 R23

$$\begin{array}{r} 3 \\ 77 \\ \underline{\times 5} \\ 385 \end{array}$$

Long division is <u>self-correcting</u> – can always subtract a bit more:

<u>Ex 6:</u>

$$\begin{array}{rl}
 & 4 \\
 & \underline{33} \\
26 & \overline{)1138} \\
 & \underline{-78} \\
 & 35 \quad \longleftarrow \text{check – too big!} \\
 & \underline{26} \\
 & 98 \\
 & \underline{78} \\
 & 20
\end{array}$$

Ans. 43 R20

<u>Ex 7:</u> (individual 1 min) Finish

$$\begin{array}{rl}
 & 5 \\
43 & \overline{)2594} \\
 & \underline{-215}
\end{array}$$

$\boxed{\text{HW}}$ Do Problem Set 15, Bring textbook & PM 6A

What to bring to class:
Ask students to bring PM 4A and 5A.

4.1 – Prealgebra

* Algebra is generalized arithmetic – we just use letters as names for numbers and rearrange as before.
* In school algebra <u>evolves</u> from arithmetic.
    * <u>Say/Discuss:</u>
        * Algebra isn't a new or different subject! Just quicker & more flexible way to do arithmetic.
        * Sometimes students see no algebra until grade 7 or 8 and it is introduced as a new subject (was it that way for you??) Better to slowly introduce use of letters in arithmetic problems in elementary school.

Use of letters ("prealgebra") not hard:

<u>EX 1</u>:

$$\left.\begin{array}{l} 7 + \_\_ = 12 \\ 7 + ? = 12 \\ 7 + x = 12 \quad \text{what is } x? \end{array}\right\}$$

clearly the same.
say: can easily be understood by elementary students.

<u>Ex 2</u>: (Russian grade 2) What do the letters stand for?

$k - 17 = 28$

$45 \div c = 5$

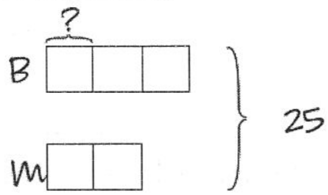
say: not always "x"

Also gives an alternate way to solve problems.

<u>Ex 3</u>: Kate bought 3 books & a magazine for a total of $25.
If the magazine cost twice as much as each book, find the cost of one book.

2 Teacher Solutions:

$\boxed{\text{Using diagrams}}$



5 units = 25

1 unit = 5

Each book cost \$5.

Using algebra

Let b = cost of books in dollars ← units specified, so letter in number NOT – number w/ units

meaning of letter clearly identified

cost of magazine = $2b$

cost of 3 books = $3b$

Total

$$\left.\begin{aligned} 3b + 2b &= 25 \\ 5b &= 25 \\ b &= 5 \end{aligned}\right\} \text{steps clearly shown in order}$$

Each book cost \$5. ← answer statement.

say: You will be asked to give such alg. solutions on HW – this is what I expect.

Compare: *same reasoning ($b \leftrightarrow$ 1 unit)

say: diagram is easier for problem with small numbers, but if you had 130 books & 48 mag's the diagram is harder & alg. the same

Caution: Letters stand for numbers.

3 quarts – quantity

number: 3; unit: quarts

say: In word problems (& real life) we usually deal with quantities = number & unit. Letters stand for numbers, so you must still specify the unit.

Ex: He drank x cups of water —— good
He drank x water ———— bad

Expressions:

Have students read def's on pg 89 EMT

Note: *Like complete sentences

*notation change: "3 times x" now $3 \cdot x$ or $3x$

*key feature: expression can be evaluated by replacing letters by numbers.

$$3x + 5 \rightsquigarrow 17$$

let $x = 4$

Teach expressions by:

* building expressions
* word problems
* simplifying & rearranging

go back & forth

Evaluating

PM 6A pg 6

*Go over pg 6 – example of building an expression.

*Read HW problem #9 set 16. (Will start it now)

PM pg 7

1. a) 13	b) $x + 8$	B		this is how
2. a) $10 - 2 = 8$	b) $m - 2$	B		HW should
3. a) $w - 5$kg	b) $8 - 5 = 3$	E		look.

*Assign 4 – 9 to each table of students. Give 2 min.
Call on students for answers.

Ask: How many different letters used? Any letter can represent a number.

Note: notation changes $6 \times 3 \longrightarrow 6 \cdot 3$ & $6c$

$x \div 3 \longrightarrow x/3$

students now know fractions. (we will in ch 6)

4 ways to build expressions:

1) Tables – pg 6 & 8 PM 6A

Caution: Not all tables lead to expressions

Ex

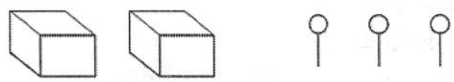
Time	Rainfall rate
11 am	1/12" per hour
12 pm	1" per hour
1 pm	1 1/2" per hour
2 pm	______

No formula!

say – other examples, stock market, gas prices, can make table, but don't lead to algebra!

2) Set Model:

say: models which worked for whole numbers still work for expressions.

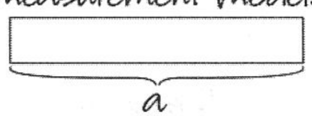


n lollypops in each box

Total number of lollypopps: $2n + 3$

See: PM 6A pg 10 #10

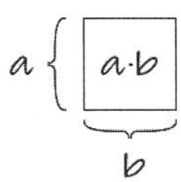
3) Measurement Model:



or scales (weight)

see PM 6A pg 7 #3

4) Rectangular Array:



(Grade 6: Students know area = product)

$\boxed{\text{HW}}$ #16 #3, 6, 9 (different from syllabus)

bring PM 6A

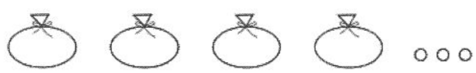
What to bring to class:
Ask students to bring PM 4A and 5A.

4.1 – Prealgebra – cont.

<u>Last time</u>: * Letter represents number
* Expressions – built by: building expressions
word problems
simplify and rearrange
* Build by: Table , Models

<u>Arithmetic with Expressions</u>:

<u>Ex</u>: m = # of marbles in a bag

 $\longleftarrow 4m + 3$

<u>Add</u>:  $\longleftarrow 2m + 1$

$$\begin{aligned} 4m + 3 + 2m + 1 &= 4m + 2m + 3 + 1 \\ &= (4 + 2)m + 3 + 1 \\ &= 6m + 3 + 1 \\ &= 6m + 4 \end{aligned}$$

what property?

<u>Subtract</u>: Go over PM 6A pg 13 #19

So: separately add terms involving m & those with no m

Say: similar to "add ones & tens separately"

Don't need pictures:

$$\begin{aligned} 4k + 9 - 3k + 4 &= (4k - 3k) + (9 + 4) \\ &(4 - 3)k + 13 \\ &1k + 13 \\ &k + 13 \end{aligned}$$

Call on students to answer questions in 1st column of problem 21 (PM 6A p 13)

* increased complication
* no pictures

<u>Def</u>: An <u>equation</u> is a statement that two expressions are equal.

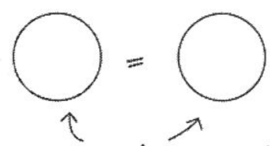
<u>Ex</u>: $12x - 3 = 33$ can solve

$x2 + y2 = 22$ can't solve

<u>Prereq</u>: Meaning of "=" (say: seems obvious but often misunderstood)

* Diagnostic test: $3 + 9 = \_\_\_ + 8$

common answer 12 for students who think "=" means "compute" NO!
(say: comes from by seeing problems ending in "=")



two sides are the <u>same number.</u>

<u>Teaching Remark</u>: Never "run equality signs"

$3\cdot 4 + 8 - 2$

$\cancel{3\cdot 4 = 12 + 8 = 20 - 2 = 18}$

Recopy: $3\cdot 4 + 8 - 2 = 12 + 8 - 2 = 20 - 2 = 18$

<u>Types of Equations</u>

(1) In $x + 3x = 96$ we can solve for x.

(2) $y = t + 3$ cannot be solved, shows relationship between t & y

(3) $4m - m = 3m$ true for all values of m, called <u>identities</u>

<u>Remaining time:</u>

(1) Bargraph activity

(2) Do Mental Math 1 & 2 from HW #16

$\boxed{\text{HW}}$ finish HW set 16.

What to bring to class:
Ask students to bring PM 4A and 5A.

4.2 – Identities, Properties, Rules

Algebraic Identities are equations which are true no matter which numbers their letters represent.

$$3x + 8x + 5 - 2 = 11x + 3$$

Teaching Sequence: Ex: Commutative Prop

1) Principle – we can add in either order
2) Examples – $3 + 5 = 5 + 3$ etc
3) Precise Statements – $a + b = b + a$ for whole #'s a + b.

– without algebra we cannot say exactly what we want to.

say: algebra is sometimes needed as a language to talk about arithmetic; necessary teacher knowledge.

Arithmetic Properties: For any whole numbers a, b, c

1) Commutative $\Rightarrow a + b = b + a$ or $ab = ba$
2) Associative $a + (b + c) = (a + b) + c$ or $a(bc) = (ab)c$
   (1) & (2) any order property, but no precise way to say
3) Distributive $a(b + c) = ab + ac$
4) Additive & Multiplicative Identities: $a + 0 = a$
   $a \cdot 1 = a$

these are still statements about numbers!

Arithmetic Properties are foundational identities:

* describe basic ways numbers behave
* all other identities can be derived from them.

From Arithmetic Prop get:

1) <u>Rules</u> = identities so simple & useful that they are worth memorizing.

say: "Rule" means "without exception" not "prescribed law"

Ex: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$ "invert & multiply" (ch 6)

$(-a)(-b) = ab$ (ch 8)

2) <u>Others</u> – not worth memorizing

Ex:
$$\begin{aligned} 6k(5x+3) + 2kx &= 30kx + 18k + 2kx && \text{DP} \\ &= 30kx + 2kx + 18k && \text{Comm} \\ &= (30+2)\,kx + 18k && \text{DP} \\ &= 32kx + 18k \end{aligned}$$

In prealgebra identities are obtained from:

* models

try it!

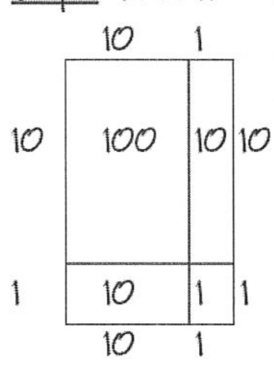
* Examples $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc} \longleftrightarrow \frac{1}{3} \div \frac{4}{7} = \frac{7}{3.4}$

generalize it.

* derived from properties.

<u>Ex:</u> $(a+b)^2$

<u>Step 1</u>: Find $11^2 = (10+1)^2$



$11^2 = (10+1)^2 = 100 + 2 \cdot 10 + 1 = 121$

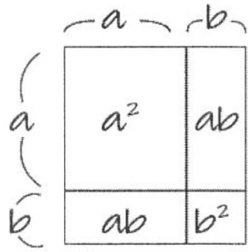
Similarly:

$21^2 = (20+1)^2 = 400 + 2 \cdot 20 + 1 = 441$

$41^2 = 1600 + 2 \cdot 40 + 1 = 1681$

$61^2 =$ ____.

Step 2: Find $(a+b)(a+b)$



$$(a+b)(a+b) = (a+b)a + (a+b)b \qquad \text{DP}$$
$$= a^2 + ba + ab + b^2 \qquad \text{DP}$$
$$= a^2 + 2ab + b^2 \qquad \text{CP}$$

agree!

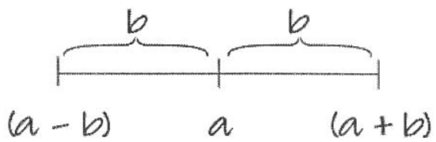
Mental Math $\quad (32)^2 = (30+2)^2 = 900 + 2\cdot 60 + 4 = 1024$

$(24)^2 = 400 + 2\cdot 80 + 16 = 576$

students try
30 sec.

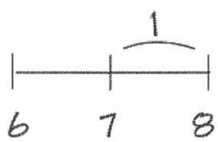
Ex 2: $(a+b)(a-b) = a^2 - b^2$

Mental Math use: Given $a+b$ & $a-b$



average = a
distance from average = b

Ex:



$6 \times 8 = 7^2 - 1^2 = 49 - 1 = 48$

average of 6 & 8 $\quad$ distance from average

$9 \times 7 = 8^2 - 1^2 = 63$

$8 \times 12 = 10^2 - 2^2 = 96$

$14 \times 16 = 15^2 - 1^2 = 225 - 1 = 224$

$38 \times 42 = 40^2 - 2^2 = 1600 - 4 = 1596$

$13 \times 17 = 15^2 - 2^2 = 221$

Special case of "double distributive property"

Ex: $(a+b)(c+d) = (a+b)c + (a+b)d$ DP

$= ac + bc + ad + bd$ DP

Do <u>not</u> use "FOIL" (say: first, outer, inner, last)

Does not generalize $(a+b+c)(x+y) = ?$

Students should learn Distributive Property!

$\boxed{\text{HW}}$ Read 4.2, Do HW #17

What to bring to class:
Ask students to bring PM 4A and 5A.

4.3 – Exponents

Write $2^n = \underbrace{2\cdot 2\cdot 2\cdots 2}_{n \text{ times}}$

say: Just notation!

<u>Ex</u>: $2^5 = 32$
$2^8 = 256$
$2^{10} = 1024$

<u>Def</u>: $\overset{\text{exponent}}{\underset{\text{base}}{a^n}} = \underbrace{a\cdot a\cdots a}_{n}$ for a, n positive whole numbers.

<u>Consequences of Definition</u>:

<u>Ex</u>: $2^3\,2^7 = \overbrace{(2\cdot 2\cdot 2)}^{3}\cdot\overbrace{(2\cdot 2\cdot 2\cdots 2)}^{7}$
= product of 10 2's
$= 2^{10}$

$\boxed{\text{Rule 1: } a^n\cdot a^m = a^{m+n}}$

say: Just counting the number of factors

<u>Mental Math</u>: $32 \times 64 = 2^5\cdot 2^6 = 2^{11} = 2^{10}\cdot 2 = 1024\cdot 2 = 2048$

<u>Ex</u>: A germ cell divides every hour. how many cells in 36 hours?

$2^{36} = 2^{10}\cdot 2^{10}\cdot 2^{10}\cdot 2^6 = 1024\cdot 1024\cdot 1024\cdot 64 \approx 7$ billion compensation!

Ex: $2^5 \div 2^2 = \frac{2 \cdot 2 \cdot 2 \cdot \cancel{2} \cdot \cancel{2}}{\cancel{2} \cdot \cancel{2}} = 2 \cdot 2 \cdot 2 = 2^3$

$$2^{19} \div 2^{13} = \frac{\overbrace{2 \cdot 2 \cdots 2}^{19}}{\underbrace{2 \cdot 2 \cdots 2}_{13}} = \underbrace{2 \cdot 2 \cdots 2}_{6} = 2^6$$

$\boxed{\text{Rule 2} \qquad a^m \div a^n = a^{m-n} \qquad \text{when } a \neq 0, m \geq n}$

In general:

$a^m \div a^n = \frac{a^m}{a^n}$ fraction notation

$= \frac{\overbrace{\cancel{a \cdots a}}^{n} \cdot \overbrace{a \cdots a}^{m-n}}{\underbrace{\cancel{a \cdots a}}_{n}}$ def

$= \underbrace{a \cdots a}_{m-n}$ simplify

$= a^{m-n}$ def

Teaching Aside: What is $x^2 + x^3$?

Common Mistake: x^5.

In fact, doesn't simplify. Exponential rules apply only when $\times$ and $\div$ (no +, −) and are just counting factors.

Ex: $(2^3)^4 = \underbrace{2^3\ 2^3\ 2^3\ 2^3}_{12\ 2\text{'s}} = 2^{12} = 4096$

$\underline{\text{Rule 3}}$ $\quad (a^m)^n = a^{mn} \quad$ for $a \neq 0 \quad$ m, n

$$(a^m)^n = \underbrace{a^m \cdot a^m \cdots a^m}_{n\text{-times}} \qquad \text{Def}$$

$$= a^{\overbrace{m+m+m+\ldots+m}^{n}} \qquad \text{Rule 1}$$

$$= a^{mn} \qquad \text{Def of Mult.}$$

$\underline{\text{Ex}}$:

$$\begin{aligned} 2^3 \cdot 5^3 &= 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5 && \text{def} \\ &= 2 \cdot 5 \cdot 2 \cdot 2 \cdot 5 \cdot 5 && \text{CP} \\ &= 2 \cdot 5 \cdot 2 \cdot 5 \cdot 2 \cdot 5 && \text{CP} \\ &= (2 \cdot 5)(2 \cdot 5)(2 \cdot 5) && \text{Assoc.} \\ &= (2\ 5)^3 \end{aligned}$$

$\underline{\text{Rule 4}}$ $\quad a^m b^m = (ab)^m$

"When two bases have same exponent, we can form pairs"

$$a^m b^m = \underbrace{(a \cdots a)}_{m} \underbrace{(b \cdots b)}_{m} \qquad \text{def}$$

$$= \underbrace{(ab)(ab)\cdots(ab)}_{m} \qquad \text{Any - order}$$

$$= (ab)^m \qquad \text{def}$$

these 4 rules are statements about <u>numbers</u>.

<u>Ex</u>: (similar to HW)

Simplify $\quad \dfrac{2^5 \cdot 6^2 \cdot 18^2}{3^4 \cdot 4^2}$

Idea: factor into only 2's & 3's, then count up total of 2's & 3's.

$$= \frac{2^5\cdot(2\cdot 3)^2\cdot(2\cdot 3\cdot 3)^2}{3^4\cdot(2\cdot 2)^2} = \frac{2^5\cdot\cancel{2^2}\cdot 3^2\cdot\cancel{2^2}\cdot\cancel{(3^2)^2}}{\cancel{3^4}\cdot\cancel{2^4}} = 2^5\cdot 3^2 = 32\cdot 9$$
$$= 320 - 32$$
$$= 288$$

What about 5^0?

Pattern:

$$\begin{aligned} 5^3 &= 125 \\ &\quad \downarrow \div 5 \\ 5^2 &= 25 \\ &\quad \downarrow \div 5 \\ 5^1 &= 5 \\ &\quad \downarrow \div 5 \\ 5^0 &= \_ \end{aligned}$$

Guess $5^0 = 1$. Is this consistent with Rules 1 – 4?

① $5^0\cdot 5^m = 5^{0+m} = 5^m$ only if $5^0 = 1$ ✓

② $5^0 = 5^{m-m} = \dfrac{5^m}{5^m} = 1$ ✓

③ $(5^0)^m = 5^{0\cdot m} = 5^0$ 1 is only # so that $\underbrace{1\cdot 1\cdots 1}_{m} = 1 \rightarrow 5^0 = 1$.

④ $(5\cdot 7)^0 = 5^0\cdot 7^0 = 1\cdot 1 = 1$

Completely consistent! So ok to define $a^0 = 1$ for all $a \neq 0$.

What about 0^0?

Patterns:

$3^0 = 1$	$0^3 = 0$
$2^0 = 1$	$0^2 = 0$
$1^0 = 1$	$0^1 = 0$
$0^0 = \_$	$0^0 = \_$

*Suggests can't define 0^0 consistently.
*If we want $a^{m-n} = a^m \div a^n$ then $0^0 = 0^{1-1} = 0 \div 0$ undefined!

$\boxed{\text{HW}}$ Read 4.3
Do HW #18

What to bring to class:
Ask students to bring PM 4A and 5A.

5.1 – Even/Odds – Intro to Proofs

<u>Ask</u> students to write down their definition of "even number."

Explain why even + even = even using their definitions.

<u>Compile</u> list of defs

4 different definitions of <u>even</u> <u>number</u>:

a) a number which occurs by skip-counting by 2. (good to introduce)

b) an even number of objects can be paired up with no remainder (visual, used in pic proofs)

c) a number which is twice a whole number (can be represented as $2n$ for some whole #n) (general, used in alg proofs)

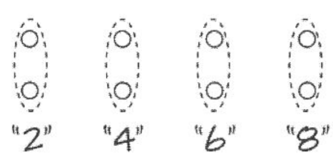
d) a number whose last digit is 0, 2, 4, 6, or 8.

say: to adults all 4 seen the same & are part of our notion of "even." But they are different ((d) depends on decimal notation). Children must learn all 4 one at a time & link them. (Teaching & math exercise)

Ex: 3574 even? check with different def's. Which is easier?

<u>Links</u>:

(a)$\Longrightarrow$(b)



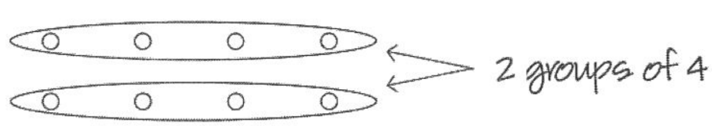
*add new pair at each step

*none left $\Longrightarrow$ even

1 left $\Longrightarrow$ odd

links def. 1 to def. 2

(b)$\Longrightarrow$(c)



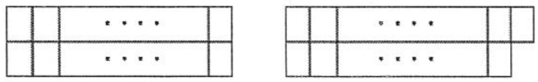
links def. 2 to def. 3

(a) ⟹ (a) count w/ no skips 1 2 3 4 doubles ⟹ skip counting

double each # $\downarrow$ 2 4 6 8

will do (d) later – see PM 4A pg25 for early connection for a ⟺ d.

<u>Array Pictures</u>: (for clearer/simpler pics)



even

⟵ meaning of dots. <u>not</u> a pic of one odd number, it is a schematic for <u>all</u> odds.

<u>Simple Proofs</u>:

<u>Def</u>: A <u>proof</u> is a detailed explanation of why a mathematical fact is true

* follow from basic rules of reasoning ⟵ same goal as teaching
* communicate – everything in math makes sense.
* <u>theorems</u> – are proven fact
* <u>Lemmas</u> – simple theorems which are used repeatedly.

Proofs start with "Proof:" and end with "□"

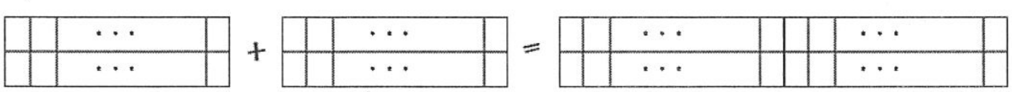
<u>Ask</u> students about their experiences in high school.
Remember proofs = explanations.

Proofs in Elementary school – informal, done with models, pics, numbers.

<u>theorem 1</u>: The sum of two even number is even.

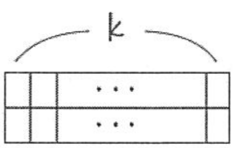
<u>Proof</u>: (picture)

explain
start w/
end w/

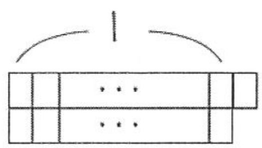


1st even # 2nd even # sum is even

□



m = 2k boxes
<u>even</u>



m = 2l + 1 boxes
<u>odd</u>

Proof (algebraic) Two even numbers can be written as

→ ask: def (c)

$2k$ and $2l$. Then

$$2k + 2l = 2(k + l) \qquad \text{Distributive Property (ask)}$$

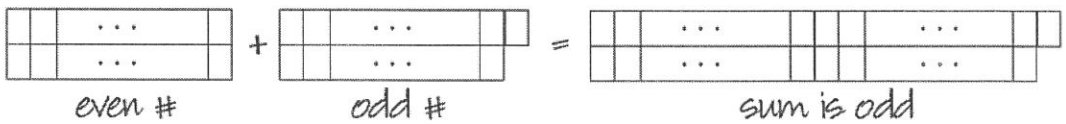
is even by def (3)

□

Theorem 2: "even + odd = odd"

The sum of an even number and an odd number is odd.

Proof: (picture)



□

Proof: (Algebraic)

Given an even number $2k$ and an odd number $2l + 1$

$$\begin{aligned} 2k + (2l + 1) &= (2k + 2l) + 1 && \text{Associative Property} \\ &= 2(k + l) + 1 && \text{Distributive Property} \end{aligned}$$

is odd.

□

* these are real proofs!

Say: Reasoning is clear. Note: Pictures become awkward when they involve large numbers or arbitrary numbers.

$\boxed{\text{HW}}$ - Read 5.1 Do HW Set #19.

What to bring to class:
Ask students to bring PM 4A and 5A.

5.2 Divisibility Test [day lecture]

All letters A, a, b, k, l, m represent whole numbers > 0.

<u>Def</u> <u>A is divisible by k</u> means k divides into A w/o remainder, i.e., there exists a quotient q such that $A = k \cdot q$.

If $A = k \cdot$ (some whole #) we can say:

* k divides A
* k is a factor of A
* k goes into A evenly
* A is divisible by k.

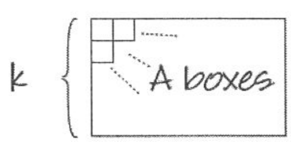
(Notation $k \mid A$ used in higher math, we do not need it however.)

[SAY: All are equivalent]

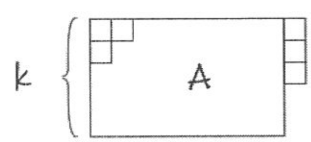
<u>Examples</u>

1. 3 divides 21
2. 75 is divisible by 5
3. 16 goes into 1024 (q = how many times $= 2^{10} \div 2^4 = 2^6 = 64$ times)
4. 3 is not a factor of 14

We can model "A is div. by k"
by



A is div. by k



A is <u>not</u> div. by k

Ex Note 16 is div. by 4. What other #'s are?

$16 + 0$	✓
$16 + 1 = 17$	×
$16 + 2 = 18$	×
$16 + 3 = 19$	×
$16 + 4 = 20$	✓
$16 + 5 = 21$	×
⋮	
$16 + B$	✓ if B is div. by 4 × if B is <u>not</u> div. by 4.

Statement (1): Suppose A is div. by k (think 16). If B is divisible by k (think 4) (2nd column) $\Longrightarrow$ (A + B) is div. by k. (3rd column)

$\Longrightarrow$ means "then" or "implies"

Now look at sums which equal 28 (which is div. by 4). When is one of the addends div. by 4, when is the other?

$28 = 0 + 28$	✓
$28 = 1 + 27$	×
$28 = 2 + 26$	×
$28 = 3 + 25$	×
$28 = 4 + 24$	✓
$28 = 5 + 23$	×
$28 = 6 + 22$	×
$28 = 7 + 21$	×
$28 = 8 + 20$	✓
⋮	
$28 = A + B$	✓ When A is, then B is too! × neither are!

(think 0.4.8 above) (think 2.8)

<u>Statement (2)</u>: Suppose A is div. by k. If A + B is divisible by k $\Longrightarrow$ B is also div. by k.

We combine statements (1) and (2) by using $\Longleftrightarrow$:

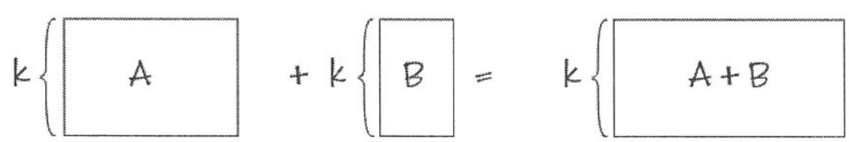
[Say: "if and only if"]

<u>Division Lemma</u>: Suppose A is div. by k. then B is div. by k $\Longleftrightarrow$ A + B is div. by k.

[Write only $\Longrightarrow$ up first, then read the last sentence backwards to get statement (2), filling in the "<" as you read it out loud.]

[SAY: For theorems with $\Longleftrightarrow$ in the statement, we must prove both directions.]

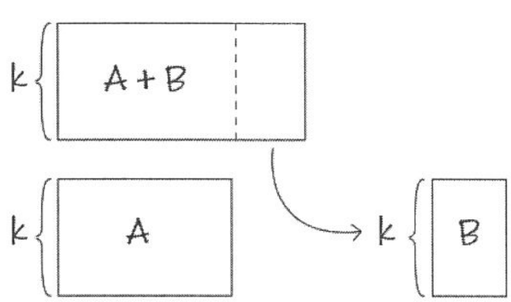
<u>Picture proof</u>: "$\Longrightarrow$" A is div. by k. B is div. by k $\Longrightarrow$ A + B is also



"A is div. by k" "B is div. by k" the sum must also!

$\square$

"$\Longleftarrow$" If A and A + B are div. by k, then B is also



B must have k rows, so it is div. by k!

$\square$

Algebraic Proof:

A is div. by k means $\quad A = k \cdot a$ for some a.

"$\Longrightarrow$" If B is div. by k, then $\quad b = k \cdot b$ for some b.

Hence $(A + B) = k \cdot a + k \cdot b = k(a + b)$ $\quad$ think = "$k \cdot$ (some whole number)"

substitution $\qquad$ distributive property

By definition $A + B$ is div. by k.

"$\Longleftrightarrow$" If $A + B$ is div. by k, then by definition $A + B = k \cdot m$ for some whole # m. then

$$B = (A + B) - A = k \cdot m - k \cdot a$$
$$= k(m - a) = k \text{ (some whole \#)}$$

Hence B is div. by k.

[When doing the algebraic Proofs, label the columns in the picture proofs by a, b, m respectively.]

$\boxed{\text{HW}}$ Read 5.2, do problems 1–5 of HW set 19.

[Day 2]

The Division is very powerful.

Ex Is 384 divisible by 6?

$$\left.\begin{array}{r}360 \text{ is} \\ 24 \text{ is}\end{array}\right\} \underset{\text{Div. Lemma}}{\Longrightarrow} 384 \text{ is!}$$

Ex Is 454 divisible by 9?

$$\left.\begin{array}{r}450 \text{ is} \\ 4 \text{ isn't}\end{array}\right\} \underset{\text{Div. Lemma}}{\Longrightarrow} 454 \text{ isn't.}$$

There are quicker ways to test for divisibility:

Divisibility Test: A number is divisible by

10	$\Longleftrightarrow$	ends with 0
5	$\Longleftrightarrow$	ends with 0, 5
2	$\Longleftrightarrow$	ends with 0, 2, 4, 6, 8
4	$\Longleftrightarrow$	last 2 digits are div. by 4
8	$\Longleftrightarrow$	last 3 digits are div by 8
3	$\Longleftrightarrow$	sum of its digits are div by 3
9	$\Longleftrightarrow$	sum of its digits are div by 9
11	$\Longleftrightarrow$	if the difference between odd-position and even-position digits is div. by 11

Examples:

1. is 8176 div. by 2? by 4? by 8?
2. is 11,265 div by 3? by 9?
3. Which divides 84,570?

   2 3 $\cancel{4}$ 5 $\cancel{8}$ $\cancel{9}$ 10

4. Is 874, 256, 921, 832 div by 3, 9?

   Tip: "Cast out 9's"

   $8\cancel{7}\cancel{4},\cancel{2}\cancel{5}\cancel{6},\cancel{9}2\cancel{1},\cancel{8}\cancel{3}2$?

   $$8+2+2=12\begin{cases}\text{div. by } 3\\ \text{not by } 9.\end{cases}$$

⑤ Is 87365 div. by 11?

$(8+3+5) = 16$

$(7+6) = \underline{-13}$

③ ⟵ not div. by 11

⟹ 87365 not div. by 11.

(but 873<u>9</u>5 is. Why?)

Why are div. test true? Use the Division Lemma.

[Write the following template on the board w/o the blanks, and doing the test for 2 at the same time.]

[Template: Use lots of space!]

Proof of the test for <u>2</u>:

By place value, any number n can be written

$n = \underline{10a + b}$

This # is <u>2 (5a)</u>, so it is div. by <u>2</u>.

This is the test #, i.e. <u>it is the last digit.</u>

By the Division Lemma,

n is div. by <u>2</u> ⟺ <u>b is div. by 2</u>

⟺ <u>b = 0, 2, 4, 6, 8</u> □

[If you have to give numerical examples along the way. For ex. write:
$124 = \underbrace{12}_{a} \times 10 + \underbrace{4}_{b}$, and so forth.]

[Tell them that all other proofs are similar.
1. Use place value to break # into the test case and a # which is div. by test #, and
2. Apply Division Lemma.]

[then do repeatedly erasing the blanks and filling in with new proof.]

Fast: Erase and fill in proofs for 5 and 10 – tell students not to copy them down.

Slow: Give time for them to recopy template, Fill in proof for test for 4.

$$n = \underbrace{100a}_{4(25a)} + \underbrace{b}_{\text{last 2 digits} < 100}$$

Fast: Same for 8

Slow: Give time to recopy template. Fill in proof for 3 (using 3 digit # abc)

$$\begin{aligned} n &= 100a + 10b + c \\ &= (99a + a) + (9b + b) + c \\ &= \underbrace{(99a + 9b)}_{9(11a+b)} + \underbrace{(a + b + c)}_{\text{sum of digits}} \end{aligned}$$

Fast: Same for 9.

Slow (very slow)

Test for 11, n with digits a b c d

$$\begin{aligned} n &= 1000a + 100b + 10c + d \quad \text{P.V.} \\ &= (1001a - a) + (99b + b) + (11c - c) + d \\ &= \underbrace{(1001a + 99b + 11c)}_{11(91a + 9b + c)\text{ is div. by 11}} + \underbrace{(-a + \underline{b} - c + \underline{d})}_{\text{test case}} \end{aligned}$$

test case

$(b + d) - (a + c)$

"odd" – "even"

$\boxed{\text{HW}}$ Read §5.2 again, do the rest of HW set 19 [do not turn in previous work]

What to bring to class:
Ask students to bring PM 4A and 5A.

5.3 Factors and Primers

A <u>factorization</u> of a number n is a way of writing it as the product of 2 or more numbers:

$$n = \underbrace{a \times b}_{\text{factors}}$$

<u>EX</u>

$$\begin{aligned} 12 &= 3 \times 4 \\ &= 2 \times 6 \\ &= 2 \times 3 \times 2 \\ &= 12 \times 1 \end{aligned}$$

All factors of 12

$\boxed{1, 2, 3, 4, 6, 12}$

Every whole number n has "trivial" factors 1 and n.

<u>Def</u> A whole number $n > 1$ is <u>prime</u> if its only factors are 1 and n. If it has at least one other factor it is <u>composite</u>.

[Say: 0 and 1 are neither prime nor composite]

[Do some examples of prime and composite]

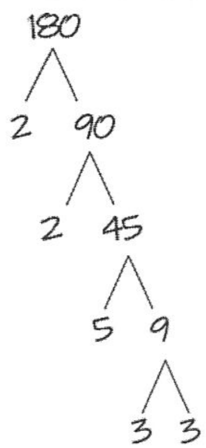
To find primes, use the "Sieve of Eratosthenes" (Era – toss – thin – e's)

[First person to estimate the Earth's circumference, tilt, size, and distance from earth to the sun and moon. – 3rd century BC]

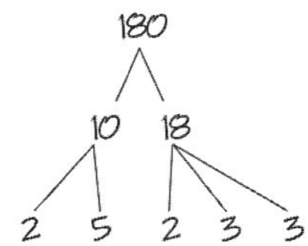
	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~	11	~~12~~
13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~	~~21~~	~~22~~	23	~~24~~
~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~	31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~

[Note after circling 5 and crossing out multiples, the rest are prime. Important for HW]

Repeated factoring gives a factor tree



- or -

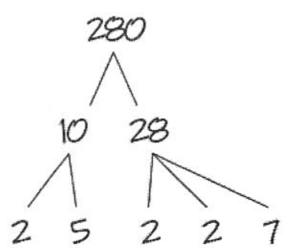


Tip: Split by large factors!

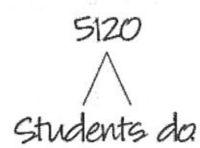
Going as far as possible yields prime factorization – a factorization as a product of primes.

Written $n = p_1 \cdot \ldots \cdot p_k$ (or $n = p_1$ when n is prime)

Ex

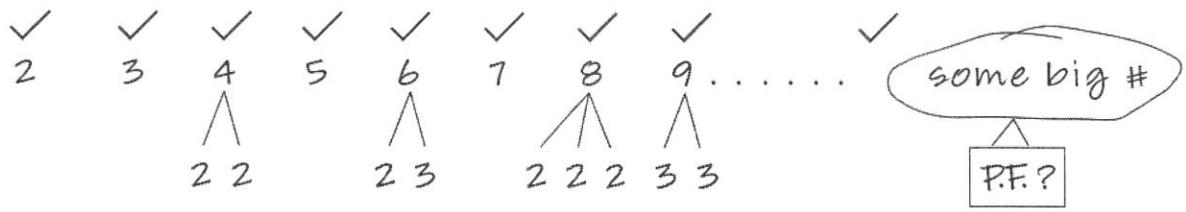


31
prime



Fundamental theorem of Arithmetic. Every whole number (except 0, 1) is either prime or a product of primes. Furthermore, each whole # has only one prime factorization.

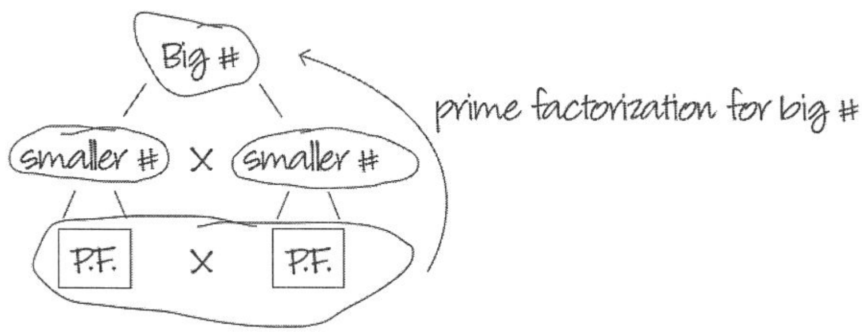
Elementary School Proof Suppose we listed every # up to some really large number and found that they all had prime factorizations.



Does that large number also have a prime factorization?

- If it is a prime – no problem, put it on our list and look at the next larger #
- If not it is the product of 2 smaller numbers – which are on our list – each have prime factorizations.

Multiplying the two prime factorizations gives a prime factorization for the large number.



In either case we can put that large # on our list and look at the next number. By the same argument, we can put that # on our list as well. [Explain why]

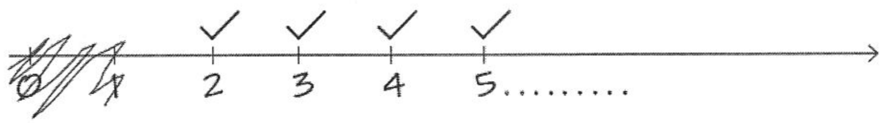
Each time we put a number on our list, it helps us to explain why we can put the next number on the list as well.

Thus our list grows until it contains every whole number!

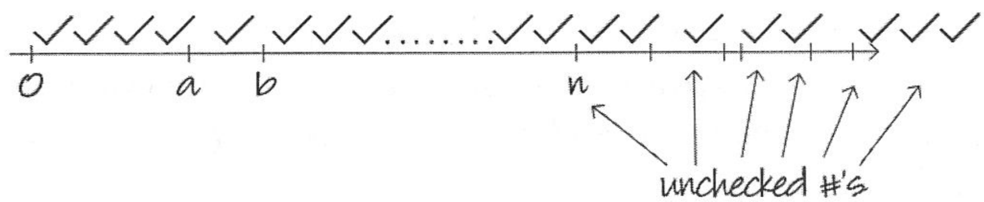
□

Proof (using different logic)

Walk along the # - line. Put a ✓ over each number which is prime or a product of primes.



Suppose some #'s can't be checked Let n be the samllest of those numbers.



n is not prime $\Longrightarrow$ n is composite and must factor: $n = a \cdot b$ where a and b are smaller and checked Each can be written as a product of primes. $\Longrightarrow$

$$n = a \cdot b = (\text{product of primes}) \cdot (\text{product of primes})$$

n can be checked, but we said it couldn't be!

Contradiction (something is wrong)

The only possibility: Our assumption that some #'s can't be checked is wrong $\Longrightarrow$ all of them can be!

□

HW Read § 5.3 and do HW set 20.

What to bring to class:
Ask students to bring PM 4A and 5A.

5.4 More Primes

[Say: In todays class we put together the divisibility test with the Fundamental theorem of Arithmetic (FTA)]

Test Primality

To test if a number n is composite or prime:

1st: Need only check for prime factors.

If n is composite, $n = P_1 P_2 \ldots P_k$ (by FTA)

$= P_1 \cdot$ (some whole #)

$\Rightarrow n$ has prime factor.

[Say: this is obvious, if n is divisible by 6, then it must be divisible by 2 and 3.]

2nd: Need only check primes up to $\sqrt{n}$.

Ex Factors of 36: $1 \quad \underbrace{2 \quad 3}_{\text{prime factors of 36}} \quad 4 \quad \underset{\sqrt{36}}{\overset{6}{\parallel}} \quad 9 \quad 12 \quad 18 \quad 36$

$2 =$ smallest P.F. $\leq \sqrt{36}$

Ex What is 13×17? 221 MM: $15^2 - 2^2$

Factors of 221: 1 13 17 221

$13 =$ smallest P.F. $\leq \sqrt{221} \approx 15$

Guess: $1 < \text{smallest P.F.} \leq \sqrt{n}$

Check: $n = P_1 \cdot P_2 \cdot P_3 \cdots P_k$ by FTA

Assume P_1 is smallest P.F.

then $n = P_1 \cdot P_2 \cdots P_k \geq P_1 \cdot P_2 \geq P_1 \cdot P_1$

$\Rightarrow n \geq P_1^2 \Rightarrow \sqrt{n} \geq P_1$ ✓

$\Rightarrow$ At least one factor of a composite # must be $\leq \sqrt{n}$. If not, n is prime.

Primality Test: To test if a number n is prime, one need only search for prime factors p of n with $p \leq \sqrt{n}$.

Ex Is 163 prime? $\sqrt{163} \leq \underbrace{\sqrt{169}}_{\text{Estimate}} = 13$

$\cancel{2}$ $\cancel{3}$ $\cancel{5}$ $\cancel{7}$ $\cancel{11}$ $\cancel{13}$

Div. test: 2, 3, 5

7: 140, 23, No

11: 110, 53, No

13: 169 − 6, NO

$\boxed{\text{Yes 163 is prime}}$

Ex Is 1001 prime? $\sqrt{1001} \leq \sqrt{1032} = \sqrt{2^{10}}$

$= 2^5 = 32$

$\cancel{2}$ $\cancel{3}$ $\cancel{5}$ 7 11 13 17 19 23 29 31

D.T.: 2, 3, 5

7: 700 ✓

280 ✓

$\underbrace{21}_{\text{Division Lemma.}}$ ✓

No. 1001 is not prime!

the number of primes.

Have students do the following exercise: [3 – 4 min]

Create a handout (using Mathematica, etc.) of all primes up to 2000. Ask them to count the # of primes in each of the intervals: 0 – 250, 251 – 500, 501 – 750, 751 – 1000, 1001 – 1250, 1251 – 1500, 1501 – 1750, 1751 – 2000.

Have them generate this table:

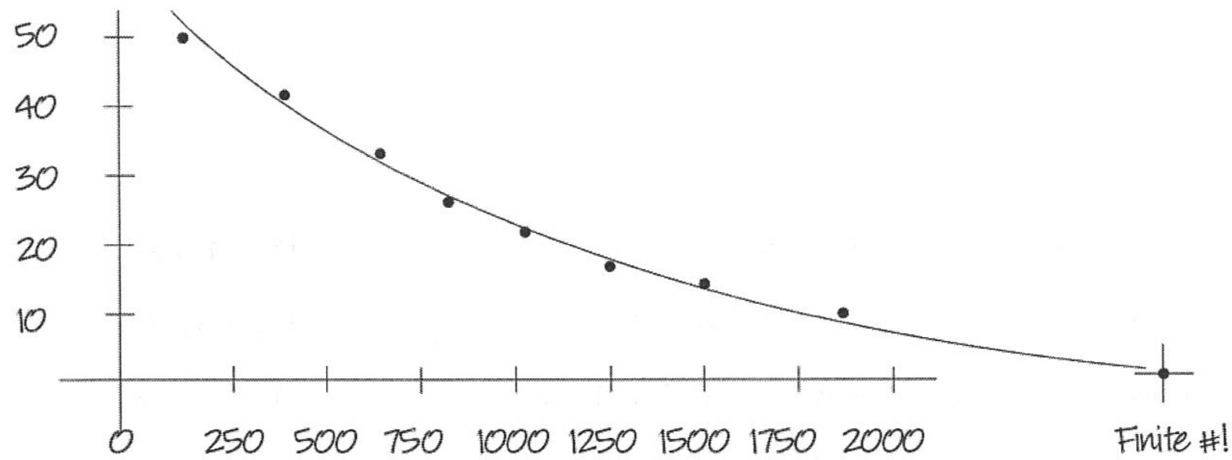
Interval	# of primes
0 – 250	50
251 – 500	42
501 – 750	37
751 – 1000	36
1001 – 1250	36
1251 – 1500	35
1501 – 1750	33
1751 – 2000	31

Ask:

- Is the # of primes increasing or decreasing?
- Make a conjecture about the # of primes.

the idea, of course, is to get them to say that there are a finite # of primes.

then draw:



[Say: Furthermore, HW 4 in todays HW shows that for any # n, there is n consecutive #'s which are not prime. there seems to be a lot of evidence for a finite number of primes]

Tell students to put space here . . .

<u>Conjecture</u>: there are a finite # of primes.

If so list them in order:

2, 3, 5, 7, 11, ... P

greatest prime.

and consider $N = (2 \cdot 3 \cdot 5 \ \ldots\ldots\ P) + 1$

By FTA, since $N > 1$ it is prime or a product of primes. But it can't be prime since $N > P$.

largest prime

Since it is a product of primes, some prime $p > 1$ in the list $2, 3, \ldots, P$ must divide N.

Since $2 \cdot 3$ .... P and N are div by P, the division lemma $\Rightarrow$ 1 is div. by P.

Contradiction because $p > 1$.

[Say: Our conjecture must be wrong! The opposite must be true]

There are an infinite # of primes.

[Now go back and fill in "theorem [Euclid]: there are an ......" before conj. erase "conjecture:" and replace with "Proof: Suppose," then put box $\square$.

This type of proof is called "Proof by contradiction." It is useful in teaching as well:

[Say:

Jimmy states "$(a+b)^2 = a^2 + b^2$"

Tell Jimmy, if this is true, then

$$\begin{array}{l} (6+7)^2 = 6^2 + 7^2 = 36 + 49 = 85 \\ \quad \| \\ 13^2 = 169 \end{array}$$

But $\qquad +1 \qquad$ Contradiction!

Something must be wrong with your formula!]

$\boxed{\text{HW}}$ Read § 5.4 Do HW set 21.

List of primes up to 2000

2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71
73	79	83	89	97	101	103	107	109	113	127	131	137	13	149	151	157	163	167	173
179	181	191	193	197	199	211	223	227	229	233	239	241	251	257	263	269	271	277	281
283	293	307	311	313	317	331	337	347	349	353	359	367	373	379	383	389	397	401	409
419	421	431	433	439	443	449	457	461	463	467	479	487	491	499	503	509	521	523	541
547	557	563	569	571	577	587	593	599	601	607	613	617	619	631	641	643	647	653	659
661	673	677	683	691	701	709	719	727	733	739	743	751	757	761	769	773	787	797	809
811	821	823	827	829	839	853	857	859	863	877	881	883	887	907	911	919	929	937	941
947	953	967	971	977	983	991	997	1009	1013	1019	1021	1031	1033	1039	1049	1051	1061	1063	1069
1087	1091	1093	1097	1103	1109	1117	1123	1189	1151	1153	1169	1171	1181	1187	1193	1201	1213	1217	1223
1229	1231	1237	1249	1259	1277	129	1283	1289	1291	1297	1301	1303	1307	1319	1321	1327	1361	1367	1373
1381	1399	1409	1423	1427	1429	1433	1439	1447	1451	1453	1459	1471	1481	1483	1487	1489	1493	1499	1511
1523	1531	1543	1549	1553	1559	1567	1571	1579	1583	1597	1601	1607	1609	1613	1619	1621	1627	1637	1657
1663	1667	1669	1693	1697	1699	1709	1721	1723	1733	1741	1747	1753	1759	1777	1783	1787	1789	1801	1811
1823	1831	1847	1867	1867	1871	1873	1877	1879	1889	1901	1907	1913	1931	1933	1949	1951	1973	1979	1987
1993	1997	1999																	

Count the number of primes in each of these intervals:

Between	Number of primes
1 – 250	
251 – 500	
501 – 750	
751 –1000	
1001 – 1250	
1251 – 1500	
1501 – 1750	
1751 – 2000	

Circle one: The numbeer of primes is __________. (increasing/decreasing).

Make a conjecture about the possible number of primes based upon your observation above. Does it look like there are infinite number of primes, or a finite number?

Conjecture: There are __________ number of primes.

What to bring to class:
Ask students to bring PM 4A and 5A.

5.5 GCF and LCM.

The greatest common factor is its own definition, i.e., list all of the common factors of 2 numbers and take the largest.

Ex GCF of 36 and 84

Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36
Factors of 84: 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84

GCF

GCF (36, 84) = 12

Easier way: Prime factor both numbers. Then pair common factors.

$$36 = 2 \cdot 2 \cdot 3 \cdot 3$$
$$84 = 2 \cdot 2 \cdot 3 \cdot 7$$

Biggest # which divides both. $= 2 \cdot 2 \cdot 3 = 12.$

The Least common Multiple (LCM) is also its own definition.

1st: Find common multiples.
2nd: Take the smallest.

Ex Find LCM of 16, 24. (Students do)

Multiples of 16: 16, 32, 48, 64, 80, 96, 112, . . .
Multiples of 24: 24, 48, 72, 96, 120, 144
smallest!

LCM (16, 24) = 48.

Easier way:

#1 Prime factorization

#2 Pair prime factors

#3 Multiply pairs with extra primes

$$\left.\begin{aligned} 16 &= \underbrace{2 \cdot 2 \cdot 2}_{\text{pairs}} \cdot 2 \\ 24 &= 2 \cdot 2 \cdot 2 \cdot 3 \end{aligned}\right\} \text{extra's}$$

$$LCM(16, 24) = \underbrace{2 \cdot 2 \cdot 2}_{\text{pairs}} \cdot \underbrace{2 \cdot 3}_{\text{extra}} = 48$$

Note: $GCF(a, b) \leq \min(a, b)$

$LCM(a, b) \geq \max(a, b)$

In fact, one can define GCF and LCM in terms of prime factorization.

Prime factorizations of $a = p_1^{r_1} \cdot p_2^{r_2} \cdots\cdots p_k^{r_k}$

$b = p_1^{s_1} \cdot p_2^{s_2} \cdots\cdots p_k^{s_k}$

[SAY! P's all different from each other, but same for a and b.]

then $GCF(a, b) = p_1^{\min(r_1, s_1)} \cdot p_2^{\min(r_2, s_2)} \cdots\cdots p_k^{\min(r_k, s_k)}$

$LCM(a, b) = p_1^{\max(r_1, s_1)} \cdot p_2^{\max(r_2, s_2)} \cdots\cdots p_k^{\max(r_k, s_k)}$

Ex Find GCF & LCM of 36 and 84

$36 = 2^2 \cdot 3^2 \cdot 7^0$

$84 = 2^2 \cdot 3^1 \cdot 7^1$

means we put $7^0 = 1$ to get same # of primes

$$GCF(36, 84) = 2^{\min(2,2)} \cdot 3^{\min(2,1)} \cdot 7^{\min(0,1)}$$
$$= 2^2 \cdot 3^1 \cdot 7^0 = 4 \cdot 3 = 12 \checkmark$$

$$LCM(36, 84) = 2^{\max(2,2)} \cdot 3^{\max(2,1)} \cdot 7^{\max(0,1)}$$
$$= 2^2 \cdot 3^2 \cdot 7^1 = 4 \cdot 9 \cdot 7 = 9 \cdot 28 = 280 - 28 = 252$$

Hw Problem in 5b in Hw set 22:

Prove that $GCF(a, b) \cdot LCM(a, b) = a \cdot b$.

Note that $\min(r_1, s_1) + \max(r_1, s_1) = r_1 + s_1$. (*)

$$GCF(a, b) \cdot LCM(a, b) \overset{\text{def.}}{=} p_1^{\min(r_1, s_1)} \cdot p_2^{\min(r_2, s_2)} \cdots\cdots p_k^{\min(r_k, s_k)} \cdot p_1^{\max(r_1, s_1)} \cdots\cdots p_k^{\max(r_k, s_k)}$$

combine

$$= p_1^{\min(r_1, s_1) + \max(r_1, s_1)} \cdot p_2^{\min(r_2, s_2) + \max(r_2, s_2)} \cdot \cdots\cdots p_k^{\min(r_k, s_k) + \max(r_k, s_k)}$$

Any Order Power Rule 1.

$$= p_1^{r_1 + s_1} p_2^{r_2 + s_2} \cdots\cdots p_k^{r_k + s_k} \qquad \text{by (*)}$$

$$= (p_1^{r_1} \cdot p_2^{r_2} \cdots\cdots p_k^{r_k})(p_1^{s_1} \cdot p_2^{s_2} \cdots\cdots p_k^{s_k}) \qquad \text{Power Rule 1, Any Order}$$

$$= a \cdot b$$

Hw Problem in 5c in Hw set 22:

$$16 = 2^4 \cdot 3^0 \cdot 17^0$$
$$102 = 2^1 \cdot 3^1 \cdot 17^1$$

$GCF(16, 102) = 2^1 \cdot 3^0 \cdot 17^0 = 2$

$2 \cdot LCM(16, 102) = 16 \cdot 102 = 1600 + 32 = 1632$

$\boxed{LCM(16, 102) = 816.}$

Note: $LCM(16, 102) = 2^4 \cdot 3^1 \cdot 17^1 = 16 \cdot 51 = 800 + 16$
$= 816.$

$\boxed{\text{HW}}$ Read § 5.5 Do HW set 22

Give quiz or go over HW or practice Mental Math.

[If you want, you can show

$$GCF(a, a+b) = GCF(a, b)$$

and then go on to prove the Euclidean Algorithm.]

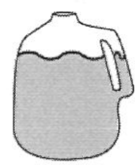
Caution – this lecture follows the book EXACTLY
be careful about boring students

Instructor – photocopy
Prim Math 2B pgs 52–57
Prim Math 3B pgs 51–62
as handout for today's class.
Students Bring PM 4A to class

Lecture 25 – Fraction Basics

Fractions used when there is a standard unit but we want to measure using (usually) smaller units called the fractional unit

Ex 4 quarts = 1 gallon
standard unit: gallon
fraction unit: quart
3 quarts = ?



Notation $\frac{3}{4}$ $\underbrace{\text{gallon.}}_{\text{standard unit.}}$

numerator = # of fractional units
denominator specifies the fractional unit; it is the number of fractional units in the standard unit.

[Say: Fractional unit usually doesn't have its own name (like "quart" above) – It is defined by the denominator.]

Notes

(1) Must know the standard unit (I have $\frac{3}{4}$ water doesn't make sense)

What to bring to class:
Ask students to bring PM 4A and 5A.

6.1 Fraction Basics

Fractions are used when there is a <u>standard unit</u>, but we want to measure using another (usually) smaller unit called the <u>fractional unit</u>

$\frac{3}{4}$ mile

<u>numerator</u> = # of fractional units; it is used to count.

<u>denominator</u> describes the fractional unit; it equals the # of fractional units in the standard one.

<u>Ex</u> 4 laps around a track = 1 mile.

std unit: mile

fractional unit: lap = $\frac{1}{4}$ mile

$\frac{3}{4}$ miles = 3 laps.

<u>Notes</u>

(1) Fractional unit usually does not have its own name (like "lap" above):
[SAY: It is defined by the denominator.]

(2) <u>Must</u> always know the std. unit. ("I have $\frac{3}{4}$ water," does not make sense.)
this notation is confusing. Common Errors:

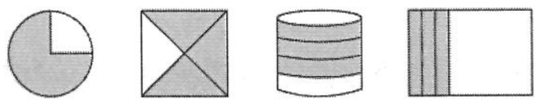
- thinking of $\frac{3}{4}$ as 2 numbers, not 1.
- thinking larger denominator means larger #.

$\cancel{(\frac{1}{5} > \frac{1}{3})}$

Teaching Sequence [SAY: Done carefully to avoid misconceptions above]:

I. (Grade 2B) Fractions are introduced informally in grades 1 – 2 using "one-half", "3 quarters" and

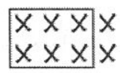
Area Models:



Not 3/4's!

Measurement Models:

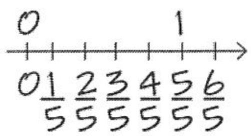
Set models (not so good)



[Have class look at pages 2B pg 52–57 in Handout (from Sing 2B).]

II. (Grade 3B) Notation with

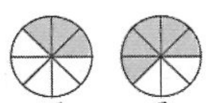
·Counting



etc.

·Comparison. Easy when

denominators same:

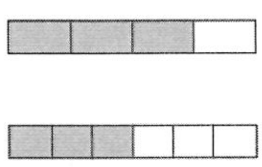


$\frac{3}{8} < \frac{5}{8}$

-or-

numerators same:

(Prepares students for $\frac{3}{x^2} > \frac{3}{x^2+3}$)



$\frac{3}{4} >$

$\frac{3}{7}$

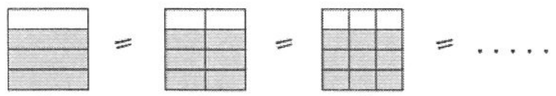
(same # of smaller units)

Prim Math 3B
pg 51–53
[Look at Handout pages <u>pg 54–56</u>]

III. (Grade 3B) Renaming fractions. Fractions can be represented in many equivalent ways (numerals)

(a) Fraction strips – [see pages <u>57–58</u> of Hand out]

(b) Subdivide areas



$$\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \cdots\cdots$$

$$\frac{3}{4} \xrightarrow[\times 2]{\times 2} \frac{6}{8}$$ illustrates

[Handout, <u>pages 38, pg 59.</u>]

$\boxed{\text{Fraction Rule 1: } \frac{a}{b} = \frac{an}{bn} \text{ for any whole \# } n > 0.}$

(c) Transition From picture ⇒ abstract

$$\frac{3}{5} = \frac{\square}{10} \; ; \; \frac{2}{3} = \frac{8}{\square}$$

IV. (Grade 4A) Simple adding and subtracting.

(a) Same denomination: [see Handout P.M. 4A pg <u>42–43</u>]

2 fifths + 2 fifths = 4 fifths

3 sevenths + 2 sevenths = 5 sevenths

<u>General Principle:</u>

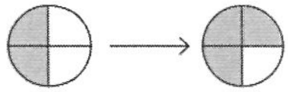
"Once you have the same fractional unit addition is the same as before."

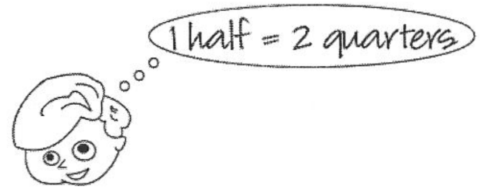
$\boxed{\text{Fraction Rule 2: } \frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}}$

(b) Closely related denominators:

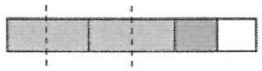
count using smallest fractional unit.

(i) $\frac{1}{2}+\frac{1}{4}=\frac{2}{4}+\frac{1}{4}=\frac{3}{4}$



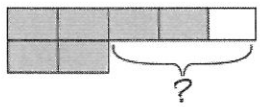


(ii) $\frac{2}{3}+\frac{1}{6}=$



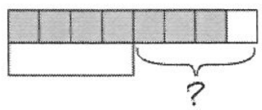
$=\frac{\square}{6}+\frac{1}{6}=\frac{\square}{6}$

(iii) $\frac{4}{5}-\frac{2}{5}$



Comparison Model.

(iv) $\frac{7}{8}-\frac{1}{2}$

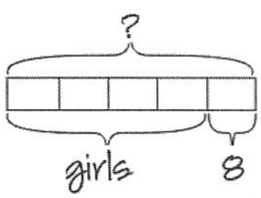


count in $\frac{1}{8}$'s!

$=\frac{7}{8}-\frac{\square}{8}=\frac{\square}{8}$

V. Word problems.

$\frac{4}{5}$ of children in a choir are girls. If 8 are boys, how many children are there altogether?



1 unit = 8
5 units = 40

there were 40 children.

Note that 1 unit = $\underbrace{\frac{1}{5}\text{ of the }\underbrace{\text{class.}}_{\text{Standard unit}}}_{\text{fractional unit}}$

$\boxed{\text{HW}}$ Read § 6.1 very carefully. [Fractions are generally a weak point for prospective teachers.]

Do HW set 24.

Bring 4A & 5A to class

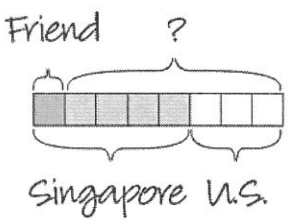
What to bring to class:
Ask students to bring PM 4A and 5A.

6.2 More Fraction Basics

Peter had 400 stamps. $\frac{5}{8}$ of them were Sinagapore stamps and the rest were U.S. stamps.

He gave $\frac{1}{5}$ of the Singapore stamps to a friend. How many stamps did he have left?

T.S.



8 units = 400
1 unit = 50
7 units = 350

He had 350 stamps.

VI. (Grade 4) Improper and Mixed Numbers. (pgs. 52–57 4A)

Def (i) A mixed number is a whole number plus a fraction: $2\frac{1}{8}$, $7\frac{3}{5}$

(Think: $2 + \frac{1}{8}$, $7 + \frac{3}{5}$) easy to understand

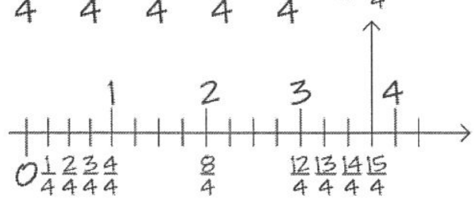
(ii) An improper fraction is a fraction $\frac{a}{b}$ with $a \geq b$. Ex. $\frac{17}{4}$, $\frac{112}{7}$

easy to calculate

Mixed numbers $\xleftrightarrow{\text{convert}}$ Improper Fractions.

Examples:

① $\frac{15}{4} = \frac{4}{4} + \frac{4}{4} + \frac{4}{4} + \frac{3}{4} = 3\frac{3}{4}$



② $4\frac{2}{3}$ is how many thirds?

14 thirds or $\frac{14}{3}$?

this is the same as $\dfrac{3 \times 4 + 2}{3} = 4\frac{2}{3}$ (+, ×)

$$= \frac{3 \times 4}{3} + \frac{2}{3}$$

$$= \frac{3 \times 4 + 2}{3}$$

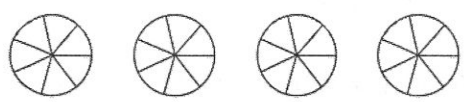
VII. Fractions from division

[SAY: Use word problems!] (5A, pages 33–35)

7 children want to share 4 cookies.

How many should each get?

[ASK: Is this partitive or measurement div.?]



Divide each cookie into sevenths ⇒ 4 whole cookies = 28 sevenths

⇒ Each child gets $4 \div 7$ = 28 sevenths $\div 7$ = 4 sevenths

$\frac{4}{7}$ cookies each!!

this implies $4 \div 7 = \frac{4}{7}$.

Hence

$\boxed{\text{Fraction Rule 3: } a \div b = \frac{a}{b}}$

this shows equivalence of the following:

$\frac{17}{3}$ ⟷ $17 \div 3$ ⟷

$$\begin{array}{r|l} & \text{5R2} \\ 3 & 17 \\ & -15 \\ & 2 \end{array}$$

⟷ $5\frac{2}{3}$ ⟷ $\frac{17}{3}$

VII. <u>Adding unlike denominators</u> (5A, pg 37-44)

We can't add fractions until we have the same fractional unit. For unlike denominators, we need to rename both fractions.

Can't add until we have same unit

1. <u>Pictoral Approach</u>

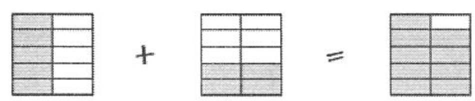
(a) $\frac{1}{2} + \frac{2}{5}$

unit $= \frac{1}{2}$ unit $= \frac{2}{5}$

Big box is Std. unit.

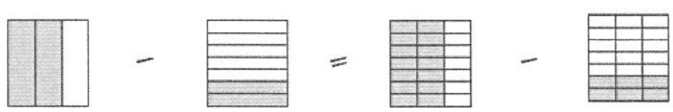
Fractional Units

Chop both ways to get common unit $\frac{1}{10}$.



$$\frac{5}{10} + \frac{4}{10} = \boxed{\frac{9}{10}}$$

(b) $\frac{2}{3} - \frac{2}{7}$ [student's do]



$$\frac{2}{3} - \frac{2}{7} = \frac{14}{21} - \frac{6}{21} = \boxed{\frac{8}{21}}$$

Note: Pictures show common unit = product of denominators.

(maybe not most efficient)

(2) Abstract Approach.

$$\frac{3}{8} + \frac{1}{6} \underset{\text{rename}}{=} \frac{9}{24} + \frac{4}{24} = \frac{13}{24}$$

LCM (8, 6) = 24.

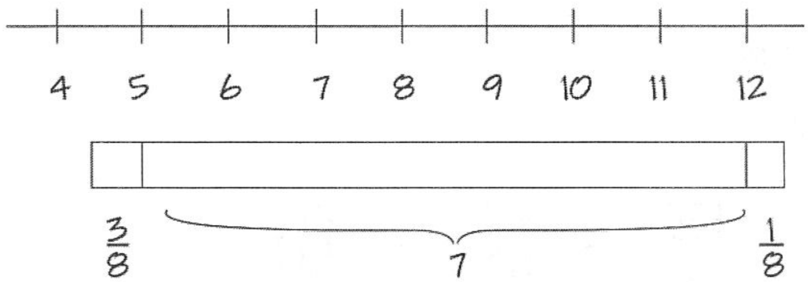
Fraction Rules 1 & 2 ⇒

$$\frac{a}{b}+\frac{c}{d}=\frac{ad}{bd}+\frac{bc}{bd}=\frac{ad+bc}{bd}$$

rule 1   same denom.   Rule 2

[SAY: Do <u>not</u> just tell students this rule! Model it until students understand, use models to develop Abstract rule]

(3) Mixed Numbers: $12\frac{1}{8}-4\frac{5}{8}$



$$12\tfrac{1}{8}-4\tfrac{5}{8}=\tfrac{3}{8}+7+\tfrac{1}{8}=7\tfrac{4}{8}=7\tfrac{1}{2}$$

<u>Common Error</u>: Beth writes $\frac{1}{3}+\frac{2}{5}=\frac{3}{8}$.

Why? [SAY: She is thinking of fractions as pairs of numbers.]

Can help Beth by.

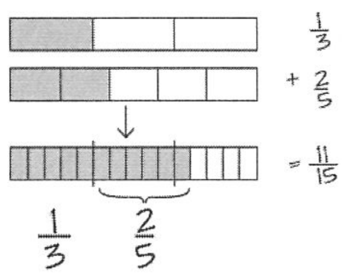
(a) "Proof by contradiction"

"Beth, your reasoning gives

$$\frac{1}{2}+\frac{1}{2}=\frac{2}{4}=\frac{1}{2}$$ Something is wrong!!"

[SAY: "Proof by Contradiction" is a teaching technique as well as a proof method!]

(b) Pictures:



(c) Only then to correct arithmetic

$$\frac{1}{3} + \frac{2}{5} = \frac{\square}{15} + \frac{\square}{15} = \frac{\square}{15}$$

Lesson: When teaching fractions, don't move pictoral ⇒ abstract too fast!

$\boxed{\text{HW}}$ Read § 6.2 very carefully! Do HW set 25

Bring SA to next class

What to bring to class:
Ask students to bring PM 4A and 5A.

6.3 Multiplication of fractions

[Do not write on board, go over in textbook:]

So far: fractions are ways to measure parts

NOw, more and more, we want to think of them as numbers which can be +, −, x, ÷.

Guiding Principles:

$$\left.\begin{array}{l}\text{Commutative}\\ \text{Associative}\\ \text{distributive}\\ \text{Identity}\end{array}\right\}\text{"How numbers behave!"}$$

For example, we could interpret fraction multiplication as:

$\frac{2}{7}\text{"x"}\frac{3}{7} = \frac{6}{7}$ (similar to how we add fractions!)

but something strange happens

$\frac{1}{3}\text{"x"}\frac{2}{3} = \frac{1 \times 2}{3} = \frac{2}{3}$

would imply $\frac{1}{3} = 1$ (since by m. ident: 1 is <u>only</u> # such that $1 \cdot a = a$)

⇒ Not a good interpretation!

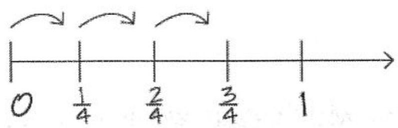
To avoid misconceptions (like the one above) teaching fractions must be done carefully!

Teaching Sequence

Case 1 whole # x fraction

old definition of x still works!

$3 \times \frac{1}{4} = 3$ groups of $\frac{1}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$



think: $3 \times \frac{2}{5} = 3$ groups of 2 fifths = 6 fifths

$= \frac{6}{5}$

Case 2 Fraction x whole.

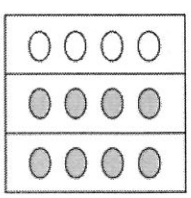
Old interpretations:

$\frac{1}{4} \times 3 = \underbrace{\frac{1}{4} \text{ groups}}_{\text{what is } \frac{1}{4} \text{ groups?}}$ of 3

$\frac{1}{4} \times 3 = \underbrace{\text{"Add 3 to itself } \frac{1}{4} \text{ times"}}_{?}$

Need a new interpretation of mult!

Ex: Note that $\frac{2}{3}$ of 12 eggs = 8 eggs.

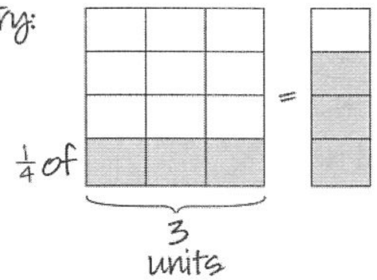


which is the same as 12 groups of $\frac{2}{3}$

$$12 \times \frac{2}{3} = \underbrace{\frac{2}{3} + \frac{2}{3} + \ldots\ldots + \frac{2}{3}}_{12} = \frac{24}{3} = 8$$

New Interpretation: $\frac{1}{4} \times 3 = \frac{1}{4}$ of 3

Try:



$\frac{3}{4}$'s !

see pages 44–45

Note: $3 \times \frac{1}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4} = \frac{1}{4}$ of $3 = \frac{1}{4} \times 3$ Commutative!

case 1   case 2

[SAY: By making this interpretation of fraction mult. we see that the commutative property still holds. If it didn't, then fractions wouldn't behave like numbers should!]

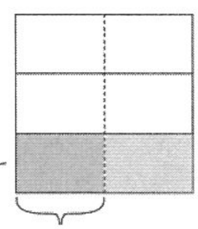
Case 3   fraction x fraction

Note: $\frac{1}{2} \times \frac{8}{9} = \underbrace{\frac{1}{2} \text{ of } 8}_{\text{reduce to case 2.}}$ ninths = 4 ninths = $\frac{4}{9}$

Start easy: $\frac{1}{2} \times \frac{1}{3} = \frac{1}{2}$ of $\frac{1}{3}$

new interpretation (case 2)

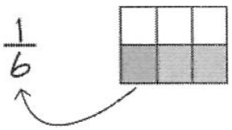
Model:



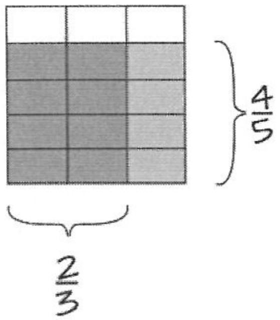
$= \frac{1}{6}$

$\frac{1}{2}$ of $\frac{1}{3}$

$\frac{1}{3} \times \frac{1}{2} = \frac{1}{3}$ of $\frac{1}{2} = \frac{1}{6}$



Ex [Students do] Model $\frac{2}{3} \times \frac{4}{5}$



$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$

numerators multiply like whole #'s: 2 x 4

new fractional unit found by multiplying denominators: 3 x 5

(See pages 49-52 pf PM 5A)

The model motivates the abstract notation:

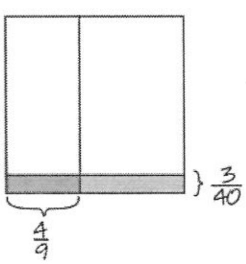
$\boxed{\text{Fraction rule 4: } \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}}$

Word Problems

skip if not enough time

Mr. Jackson brought $\frac{3}{40}$'s of a wood pile into the house to use in a fire. He used only $\frac{4}{9}$'s of what he brough in. How much of the wood pile did he actually use?

T.S.



$\frac{4}{9}$ of $\frac{3}{40} = \frac{4}{9} \times \frac{3}{40}$

$= \frac{\cancel{4} \times \cancel{3}}{{}_3\cancel{9} \times \cancel{40}_{10}} = \frac{1}{30}$

He used $\frac{1}{30}$ of the wood pile.

Division (Reviews of measurement and Partitive)

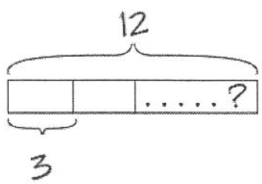
Measurement and partitive interpretations still work for fractions!

[SAY: In fact, we need them to make sense of the "invert and multiply" rule.]

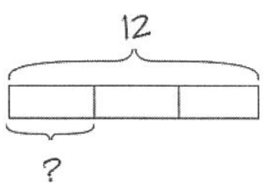
[this can't be skipped:]

[You can follow in book if out of time]

Measurement division   12 ÷ 3 means "12 is how many 3's ?"



Partitive division   12 ÷ 3 means "12 is 3 of what?"



In fractions, we use the same interpretations, but the digrams might be different.

Ex 1 (Measurement) If a road is created at $\frac{1}{2}$ miles per week, how many weeks are needed to build $1\frac{3}{4}$ miles?

SAY:

Notes: · Answer $1\frac{3}{4} \div \frac{1}{2}$.

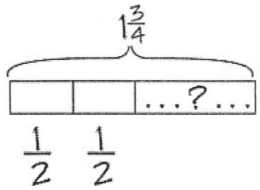
· Liping MA study: Only 43% of U.S. teachers correctly found $1\frac{3}{4} \div \frac{1}{2}$.

· Same study: only one U.S. teacher was able to make up a word problems like ex 1.

Interpretive question: $1\frac{3}{4}$ is how many $\frac{1}{2}$'s?

get students to come up with

Diagram:



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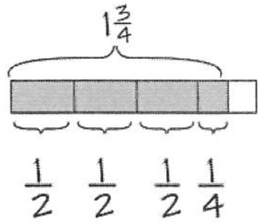


$\frac{1}{2}$ mile = 1 week

$\frac{1}{4}$ mile = $\frac{1}{2}$ week

$1\frac{3}{4}$ mile = 3 weeks + $\frac{1}{2}$ week

= $3\frac{1}{2}$ weeks

It will take $3\frac{1}{2}$ weeks.

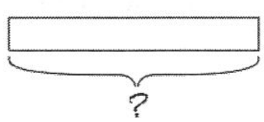
Ex 2 (Partative) If $\frac{1}{2}$ of a jump rope is $1\frac{3}{4}$ meters, what is the length of the rope?

interpretive question: $1\frac{3}{4}$ is $\frac{1}{2}$ of what?

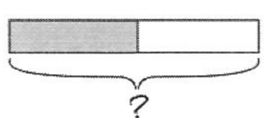
$1\frac{3}{4} \div \frac{1}{2}$.

Steps for writing down the model: [page 148 in Text book]

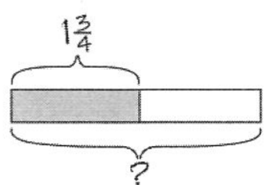
(1) Draw the "what bar" and label w/ a ?



(2) Find $\frac{1}{2}$ of it

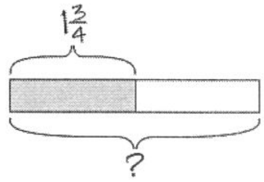


(3) Label that portion by $1\frac{3}{4}$



[SAY: $1\frac{3}{4}$ is $\frac{1}{2}$ of what? Point to each piece as you go.]

T.S.



1 unit = $1\frac{3}{4}$

2 units = $3\frac{1}{2}$

The rope is $3\frac{1}{2}$ meters long.

Note: $1\frac{3}{4} \div \frac{1}{2} = ? = 1\frac{3}{4} \times 2 = 3\frac{1}{2}$

[SAY: 1st indication we must "invert and multiply"]

$\boxed{\text{HW}}$ Read § 6.3 very carefully. Then do HW set 26.

Bring 5A & 6A to next class.

What to bring to class:
Ask students to bring PM 4A and 5A.

6.4 Dividing fractions

Ultimate goal: "invert and multiply" rule

But <u>must</u> explain concepts

- What division of fractions means
- How to divide

SAY: long time on this!

using models, interpretations, and word problems.

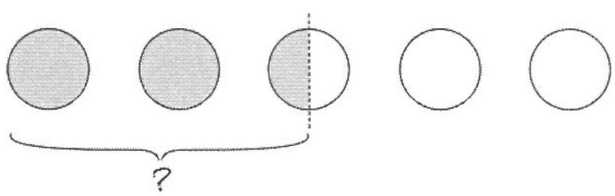
<u>Teaching Sequence</u>

<u>Case 1</u>: Whole ÷ Whole Review

<u>Ex 1</u> 2 girls share 5 cookies equally. How much did each get?

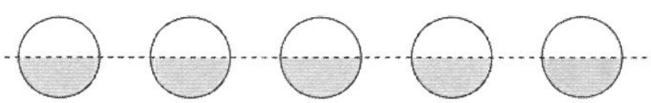
PD or MD? "5 is 2 groups of what?"

Either:



$5 \div 2 = 2\frac{1}{2}$

or



$5 \div 2 =$ shaded portion $=$ 5 half cookies $= 5 \times \frac{1}{2} = \frac{5}{2}$

Answers are equivalent: $2\frac{1}{2} = \frac{5}{2}$. But 2nd viewpoint is starting point of fraction division!

$5 \div 2 = 5 \times \frac{1}{2}$

Case 2: Fraction ÷ Whole

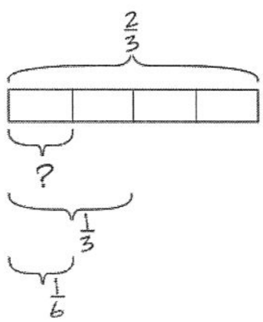
Partition fraction into groups.

Ex 2 4 boys shared $\frac{2}{3}$ quart of juice equally.

How much did each get?

PD or MD? "$\frac{2}{3}$ is 4 groups of what?"

T.S.



4 units =

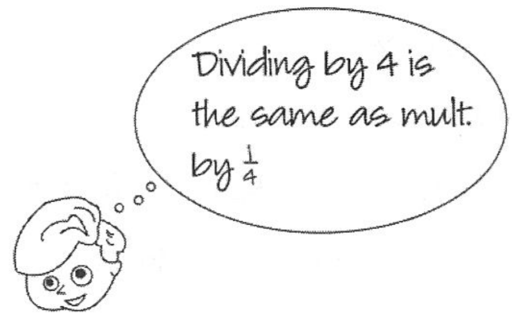
1 unit $= \frac{2}{3} \div 4 = \frac{1}{6}$.

Each got $\frac{1}{6}$ quart.

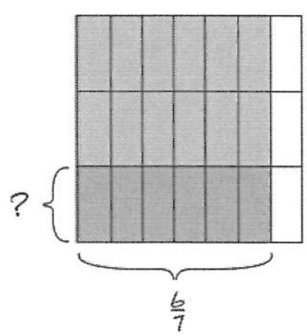
Note: $\frac{2}{3} \div 4 = \frac{1}{4}$ of $\frac{2}{3}$ (by diagram)

$= \frac{1}{4} \times \frac{2}{3}$

$= \frac{2}{12} = \frac{1}{6}$



Ex 3 $\frac{6}{7} \div 3$ using Area model.



Model shows:

$\frac{6}{7} \div 3 = \frac{6}{21} = \frac{2}{7}$

Abstractly:

$\frac{6}{7} \div 3 = \frac{1}{3}$ of $\frac{6}{7} = \frac{1}{\cancel{3}} \times \frac{\cancel{6}^{2}}{7} = \frac{2}{7}$

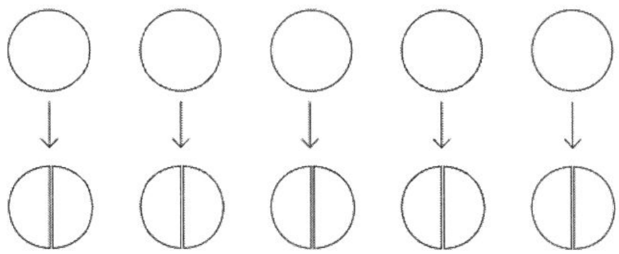
Note: $\frac{6}{7} \div 3 = \frac{6}{7} \times \frac{1}{3}$

Case 3   Whole ÷ fraction

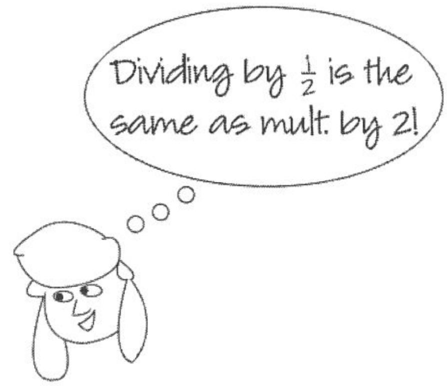
* Conceptually hardest case. Use models and word prob.

Ex 4 Jill bought 5 oranges. She cut each into $\frac{1}{2}$ pieces. How many halves did she have?

PD or MD?   5 is how many $\frac{1}{2}$'s?



10 pieces

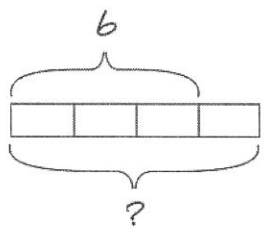


Hence $5 \div \frac{1}{2} = 10$.

Ex 5 Jim decided to walk to Jill's house from his. After 6 blocks he was $\frac{3}{4}$'s of the way. How far apart are their houses?

PD or MD?   "6 is $\frac{3}{4}$ of what?"

T.S.



3 units = 6

1 unit = 2

4 units = 8

She lives 8 blocks away.

Abstractly: $\underline{6 \div \frac{3}{4}} = (6 \div 3) \times 4 =$

$= (\frac{1}{3} \text{ of } 6) \times 4$

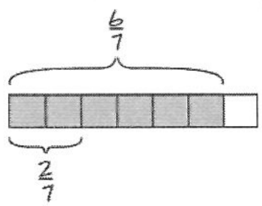
$= 6 \times \frac{1}{3} \times 4$

$= \underline{6 \times \frac{4}{3}} = \frac{24}{3} = 8.$

Case 4: fraction ÷ fraction

Ex 6 $\frac{6}{7} \div \frac{2}{7}$ using M.D.

$\frac{6}{7}$ is how many $\frac{2}{7}$'s?



Shows $\frac{6}{7} \div \frac{2}{7} = 3$

total per group # of groups

In general

$\boxed{\text{Fracion Rule 5: } \frac{a}{b} \div \frac{c}{b} = a \div c}$ (or $\frac{a}{c}$ using FR3)

Note that this leads to the common " invert and multiply" rule.

Ex $\frac{1}{4} \div \frac{2}{3}$

Abstractly: $\frac{1}{4} \div \frac{2}{3} = \frac{3}{12} \div \frac{8}{12} = 3 \div 8 = \frac{3}{8} = \frac{1}{4} \times \frac{3}{2}$

Rule 1 Rule 5 Rule 3 Rule 4

More abstractly:

$$\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bd} \div \frac{bc}{bd} = ad \div bc = \frac{ad}{bc} = \frac{a}{b} \times \frac{d}{c}$$

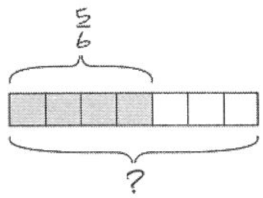
invert and multiply!

Note: $\frac{3}{2}$ is the inverse or reciprocal of $\frac{2}{3}$.

this more general rule follows from partitive interpretation:

Ex 7 $\frac{5}{6} \div \frac{4}{7}$

PD: $\frac{5}{6}$ is $\frac{4}{7}$ of what?



$4 \text{ units} = \frac{5}{6}$

$1 \text{ unit} = \frac{5}{6} \div 4 = \frac{5}{24}$

$? = 7 \text{ units} = \frac{5}{24} \times 7 = \frac{35}{24}$

<u>Abstractly:</u> $\frac{5}{6} \div \frac{4}{7} = (\frac{5}{6} \div 4) \times 7$

$= (\frac{1}{4} \text{ of } \frac{5}{6}) \times 7$

$= \frac{5}{6} \times \frac{1}{4} \times 7$

$= \frac{5}{6} \times \frac{7}{4} = \frac{35}{24}.$

$\boxed{\text{HW}}$ Read § 6.4 and § 6.5. Do HW set 27.

Bring Text book to next class!

Sing 5A

Sing 6A

What to bring to class:
Ask students to bring PM 4A and 5A.

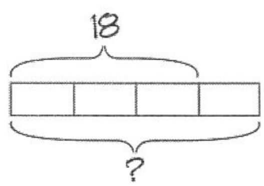
6.5 More Division of Fractions

[SAY: Today we will spend all of class on word problems. I will do a few, then you'll do some in small groups (of 2) and present them at the board.
Your HW is to write Teacher solution's to the problems in the text.
We will do some in class.]

<u>Ex</u> Sam spent $\frac{3}{4}$ of his money on an \$18 book. How much money did he have at first?

This asks "18 is $\frac{3}{4}$ of what?" $\Rightarrow$ P.D. for $18 \div \frac{3}{4}$

<u>T.S.</u>



3 units = 18
1 unit = 6
4 units = 24

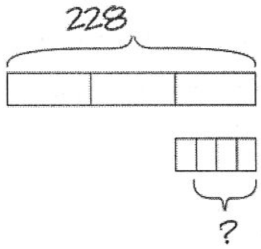
He had \$24 at first.

<u>Note</u>: $18 \div \frac{3}{4} = \overset{6}{\cancel{18}} \times \frac{4}{\cancel{3}} = 24$.

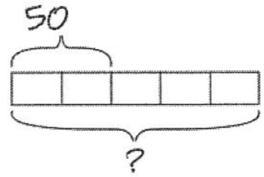
[Warm up: Assign problems (a), (c), (g), (h). From problem 5 of HW set 26. Assign 1 problem to each group of 2 students so that each group has a problem.

- While they are working, write problems on the board.
- Pick groups to give teacher's solution.
- Point out PD vs. MD for 1-step problems.

(a)

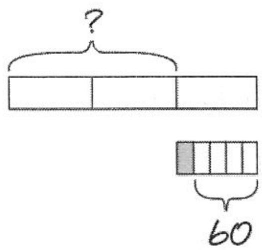


(c)

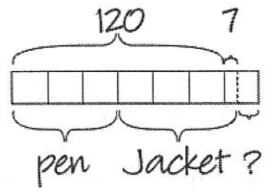


PD: "50 is $\frac{2}{5}$ of what."

(g)



(h)



He had $13 left.

6 units = 127 − 7 = 120

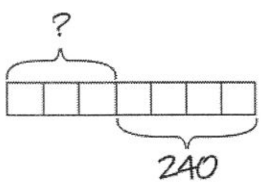
1 unit = 20

20 − 7 = $13

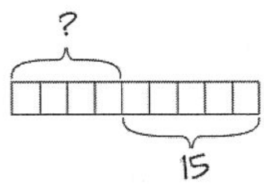
[Spend no more that 15 minutes!]

[Now assign word probs (b), (c), (f), (h) of prob 5 in HW set 27 as before.]

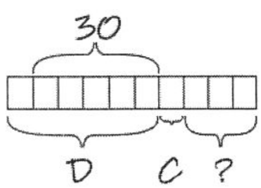
(b)



(c)



(f)



(h) [HARD!]

2 units = 8 bowls

1 unit = 4 bowls

⇒ 2 units = 6 Plates

1 unit = 3 Plates

5 units = 15 Plates

[Note: try problem (h) using algebra only before class. Enjoy!

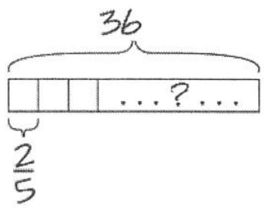
Singapore loves to give these type of problems.]

Creating word problems [SAY: U.S. teacher's are notoriously bad at this!]

Ex 1 Write a word problem using MD for $36 \div \frac{2}{5}$.

(1) MD: "36 is how many $\frac{2}{5}$'s?"

(2) Draw diagram



(3) Answer $\qquad 36 \div \frac{2}{5} = \overset{18}{\cancel{36}} \times \frac{5}{\cancel{2}} \overset{MM}{=} 90$

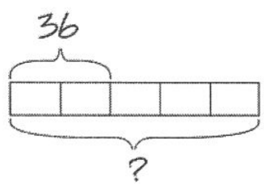
(4) Make up word problem which would produce model:

Jenny has 36 yards of ribbon. It takes $\frac{2}{5}$'s of a yard to make a bow. How many bows can she make?

Ex 2 PD word problem for $36 \div \frac{2}{5}$.

(1) PD: 36 is $\frac{2}{5}$ of what?

(2) Draw diagram:



(3) Answer: 90

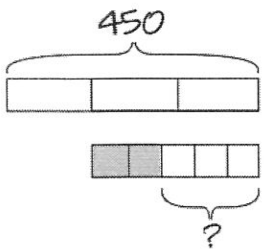
(4) Make problem. (PD works good for "before and after" situations)

John used 36 min of his break to eat lunch. If this was $\frac{2}{5}$'s of his total break time, how long is his break?

Multi step word problems

Replace step 2 with a more complicated model

HW set 27, prob 4 (c).



Solve the model: 3 units = 450 5 units = 300

2 units = 300 1 unit = 60 units

3 units = 180

Salley made 450 cookies. She sold $\frac{1}{3}$ of them and gave $\frac{2}{5}$'s of the remainder to some friends. How many did she have left?

HW Read & again. Do HW set 28.

(In problem 5g, change $\frac{2}{3} \rightarrow \frac{1}{3}$)

What to bring to class:
Ask students to bring PM 4A and 5A.

6.6 Fractions as Numbers

[SAY: Understanding fraction arithmetic becomes increasingly important as students prepare for algebra.]

Fraction aritmetic is developed using many artihmetic problems like the following:

Ex HW set 29. Prob 2d

$$\left[\left(\tfrac{1}{4}\cdot\tfrac{3}{4}\right)+\left(\tfrac{2}{3}\div\tfrac{4}{3}\right)\right]\div\tfrac{11}{12}=\left(\tfrac{1}{4}\cdot\tfrac{3}{4}+\tfrac{2}{3}\cdot\tfrac{3}{4}\right)\cdot\tfrac{12}{11} \quad \text{R5}$$

$$=\left(\tfrac{1}{4}+\tfrac{2}{3}\right)\cdot\tfrac{3}{\cancel{4}}\cdot\tfrac{\cancel{12}^{3}}{11} \qquad \text{distributive Prop.}$$

$$=\left(\tfrac{3}{12}+\tfrac{8}{12}\right)\cdot\tfrac{9}{11} \qquad \text{R1, R4}$$

$$={}_{4}\tfrac{\cancel{11}}{\cancel{12}}\cdot\tfrac{\cancel{9}^{3}}{\cancel{11}} \qquad \text{R2}$$

$$=\tfrac{3}{4} \qquad \text{R4, R1}$$

Students do rest of the problems in HW set 29, Prob 2. (in groups of 2, 5 min.)

Fraction Arithmetic – mathematics

[SAY: We used models and interpretations to generate the 5 rules of fractions.
Is it possible to use different models and interpretations to generate different (but valid) fraction rules?
Then, for instance, each country or classroom could have a different way to add fractions!]

Recall from § 4.2

"All indentities (in particular, the arith. rules) can be derived from the arith. properites."

Thus

Arith. properties $\Rightarrow$ Rules 1–5 $\Rightarrow$ fraction arithmetic.

We are <u>forced</u> to make these rules –

they <u>do not</u> depend upon the models or interpretations.

To show this, we assume there is a set called "fractions" and

- there is some way to + and × them
- they satisfy the arithmetic properties (Any order, dist, identity)

<u>The Multiplicative inverse property</u>

For each nonzero fraction x there is a unique fraction called the inverse, $\frac{1}{x}$ such that

$$x \cdot \frac{1}{x} = 1.$$

Fraction: $1, 2, 3, 4, \ldots \ldots \frac{3}{4}$

$\updownarrow \updownarrow$

Inverse: $\underbrace{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}}_{\text{fractional unit}}, \ldots \ldots \frac{1}{\frac{3}{4}}$

<u>Def</u> Each fraction can be represented by a multiple of a fractional unit:

$$\frac{a}{b} = a \cdot \frac{1}{b}.$$

To prove "properties ⇒ rules" we need the following lemma:

Lemma Assuming only arithmetic properties,

$$\frac{1}{a}\cdot\frac{1}{b}=\frac{1}{ab} \qquad \text{for } a, b \neq 0.$$

[SAY: the multiplication of 2 fractional units is again a fractional unit.]

Proof

$$\begin{aligned}
\tfrac{1}{a}\cdot\tfrac{1}{b} &= \tfrac{1}{a}\cdot\tfrac{1}{b}\cdot 1 && \text{mult. identity}\\
&= \tfrac{1}{a}\cdot\tfrac{1}{b}\cdot(ab)\tfrac{1}{ab} && \text{mult. inverse}\\
&= (a\cdot\tfrac{1}{a})(b\cdot\tfrac{1}{b})\cdot\tfrac{1}{ab} && \text{Any order}\\
&= 1\cdot 1\cdot\tfrac{1}{ab} && \text{mult. inv.}\\
&= \tfrac{1}{ab} && \text{mult. identity}
\end{aligned}$$

Thm Rules 1–5 follow from the def. of fractions and the arithmetic property.

Proof

Rule 1 $\frac{an}{bn}=\frac{a}{b}$

$$\underbrace{\tfrac{an}{bn} = (an)\tfrac{1}{bn}}_{\text{def.}} \underbrace{= an\cdot\tfrac{1}{b}\cdot\tfrac{1}{n}}_{\text{Lemma}} \underbrace{= (a\cdot\tfrac{1}{b})(n\cdot\tfrac{1}{n})}_{\text{any order}}$$

$= \frac{a}{b}\cdot 1$ def of fractions, mult. inverse

$= \frac{a}{b}$ mult. id.

Rule 2 $$\underbrace{\tfrac{a}{b}+\tfrac{c}{b} = a\cdot\tfrac{1}{b}+c\cdot\tfrac{1}{b}}_{\text{def}} \underbrace{= (a+c)\tfrac{1}{b}}_{\text{dist. prop.}} \underbrace{= \tfrac{a+c}{b}}_{\text{def.}}$$

Rule 3 $\quad a \div b = \frac{a}{b}$

def. of div.

$a \div b = x \Leftrightarrow a = bx$. Multiply by $\frac{1}{b}$

$$a \cdot \tfrac{1}{b} = \tfrac{1}{b}(bx) \underset{\text{any order}}{=} (\tfrac{1}{b} \cdot b)x \underset{\text{mult. inv.}}{=} 1 \cdot x \underset{\text{mult. id.}}{=} x$$

so $\quad a \div b = x = a \cdot \frac{1}{b} \underset{\text{def. of frac.}}{=} \frac{a}{b}$

Rule 4 $\quad$ HW

Rule 5 $\quad$ By def. $\quad \frac{a}{b} \div \frac{c}{d} = x \Leftrightarrow \frac{a}{b} = \frac{c}{d}x$.

then

$$\frac{a}{b} \cdot \frac{d}{c} = \frac{d}{c}\left(\frac{c}{d}x\right) \underset{\substack{\text{any order}\\ \text{R4}}}{=} \frac{dc}{dc} \cdot x \underset{\substack{\text{def of}\\ \text{fractions}}}{=} \left(dc \cdot \frac{1}{dc}\right)x$$

$= 1 \cdot x \quad$ mult. inverse

$= x \quad$ mult. id.

Thus $\frac{a}{b} \div \frac{c}{d} = x = \frac{a}{b} \cdot \frac{d}{c}$

Note: $\quad \dfrac{1}{\frac{a}{b}} \underset{\text{Rule 3}}{=} 1 \div \frac{a}{b} \underset{\text{R5}}{=} 1 \cdot \frac{b}{a} \underset{\text{Mult. id.}}{=} \frac{b}{a}$

"Inverses are the same as reciprocals!"

$\boxed{\text{HW}}$ Read § 6.6. Do HW set 29. Bring Primary math 5A & 6A to next 3 lectures.

What to bring to class:
Ask students to bring PM 4A and 5A.

7.1 Ratios and Proportions

GOAL: To define ratio

Have class open Sing 5A to pg 71.

Start reading together. Have students fill in blanks.

Call attention to

- Movement from Concrete ⇒ Pictorial
- Every problem is in a new setting

  page 76, simplify by crossing out

  $\cancel{4} : \cancel{10}$
  2 : 5

[AVOID WRITING RATIO IN FRACTION NOTATION!]

– not yet anyway –

- Page 77-78, note teacher's solutions
- Page 77, child hints at definition

  7 : 4 means 7 units to 4 units

  ⇒ 7 and 4 are in the same unit!

$\boxed{\text{Page 79}}$ Get student at the board!

Send 3 or 4 up to the board, assign them each 1 problem from practice 5A

Suggestion: 4, 5, 7

Give 2 minutes for student to <u>try</u> to draw teacher solutions.

Then, instructor reads problem, and class works together to build T.S. (with student at board)

read pages 80-81, note

- ratio doesn't appear to be a number.
- Do problems 4, 6, 9 of practice 5B quickly – class helps build pictures!

<u>Def</u>: A <u>proportion</u> is a statement that two ratios are equivalent

$2 : 3 = 10 : \square$

Note that

- the unit is the same for both quantities.

<u>Ex</u>: Ratio of oranges to apples is 2 : 3

really means

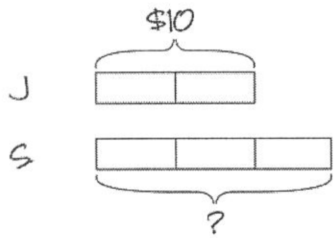
2 objects to 3 objects.

<u>Ex</u>: 50 miles to 1 hour is not a ratio, called a <u>rate</u>.

- For a specific situation, there is a unit which measures both quantities.

Ex: the ratio of Jims money to Sam's is 2 : 3.

If Jim has \$10, how much does Sam have?



2 units = \$10

$\boxed{\text{1 unit} = \$5}$

3 units = \$15

Sam has \$15.

[SAY: Here the unit is a 5 dollar bill. Jim has 2 five dollar bills, Sam has 3. 2 : 3 = 10 : 15, but each is measured w/ different units: a five vs \$1.]

Def: We say the ratio between two quantities is 2 : 3 if there is a unit so that the 1st quantity measures 2 units and the second measures 3 units.

Ratios represented as fractions

Because ratios have many equivalent representations like fractions (2 : 3 = 4 : 6 = 12 : 18 = etc. . . .), they are often written as fractions:

Turn to page 24 of Sing 6A, read and discuss problems 6–16 quickly.

Note:

- ratios can be converted into
    - fraction of total $2:3 \rightsquigarrow \frac{2}{5}$ (problem 6)
    - turned into a scale factor $2:3 \rightsquigarrow \frac{2}{3}$ (problem 7, 8)
- Fractions can be used to write an equivalent ratio (prob 9)
- After a ratio is turned into a fraction, the fraction is a number, but the ratio is not: this is because we specified a whole unit in order to write it as a fraction.

thus,

"ratios are fractions that are waiting for a standard unit to be specified."

this makes ratios more versitile then fractions, but we cant think of them as numbers.

One more time to be sure: ratios are not numbers!

If time, go over problems in Practice 3A of Sing 6A (Suggestion: 4, 7, 8)

$\boxed{\text{HW}}$ Read § 7.1 and do HW set 30

<u>Ex</u>: A bag contains 6 white and 10 red marbles. 4 white marbles and 20 red marbles were added to the bag. What is the ratio of white to red marbles in the bag?

$$\underbrace{6:10}_{\substack{\text{ratio}\\ \text{before}}} \text{ "+" } \underbrace{4:20}_{\substack{\text{ratio}\\ \text{being}\\ \text{added}}} = \underbrace{10:30}_{\text{new ratio}} = 1:3$$

correct answer!

the ratio is 1 : 3

reduce

reduce

but:

$$3:5 \text{ "+" } 1:5 = 4:10 = 2:5 \neq$$

No matter how you try you can't find a way to "add" (or −, x, ÷) Ratios are not #'s!

What to bring to class:
Ask students to bring PM 4A and 5A.

7.2 Changing ratios and intro to percents

* Do some HW probs from previous set.

Open Sing 6A to page 34-37 and discuss problems on those pages [Watch out! Think about them before going into class.]

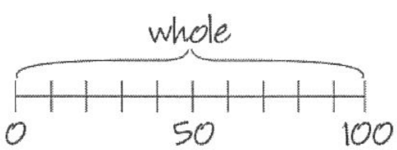
Note: We see that there are no operations (+, −, x, ÷) that take us from the before ratio to the after ratio.

In groups of 2, assign problems from practice 3C (suggestion: prob 4, 5, 7, 8)
Have students present teacher's solution at board.

<u>Percents</u>

Parts of a whole can be expressed by
- fractions
- decimals (chapter 9)
- percents

Percent means parts per hundred. "per cent" as in century or cents in a dollar.
Visualize percents:



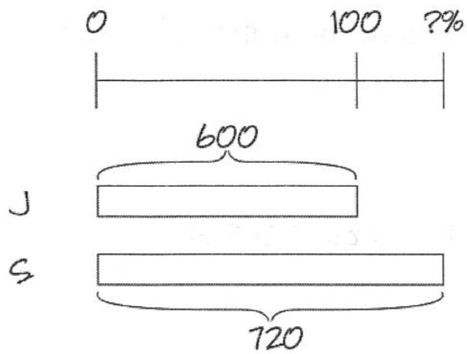
You need to know what the "whole" stands for.

Do problems 5, 6, 7 of Practice 4A in groups of 2. Teacher presents them at board.

Ex (Similar to 6A, page 55)

Jill saves \$600 and Sam saves \$720.

Express Sam's savings as a percentage $\underbrace{\text{of Jill's.}}_{\text{unit = 100!}}$



$$\$600 \longrightarrow 100\%$$

$$\div 600$$

$$\$1 \longrightarrow \frac{100}{600}\% = \frac{1}{6}\%$$

$$\times 720$$

$$\$720 \longrightarrow \cancel{720}^{\$20} \times \frac{1}{\cancel{6}}\% = 120\%$$

Sam's savings is 120% as much as Jills

Sam saved 20% more than Jill.

20% of $\underbrace{\text{Jill's money}}_{\text{whole unit}}$

Do problems 5, 7, 9, 10 of Practice 4C in groups of 2. Have students present at the board

$\boxed{\text{HW}}$ Read § 7.2 and do HW set 31.
(change) !

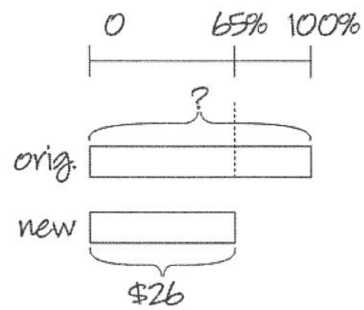
What to bring to class:
Ask students to bring PM 4A and 5A.

7.3 Solving Percent problems by the Unitary Method

"Unitary Method" = Find the whole

<u>Ex 1</u> The price of a shirt was marked down 35% to \$26. What was the original price?

<u>Start</u> ⟹
by drawing
100% scale



<u>Percent</u> \$

65% ⟶ \$26

÷13

5% ⟶ \$2

x20

100% ⟶ <u>\$40</u>

Original price is \$40.

[Instructors: Set up as

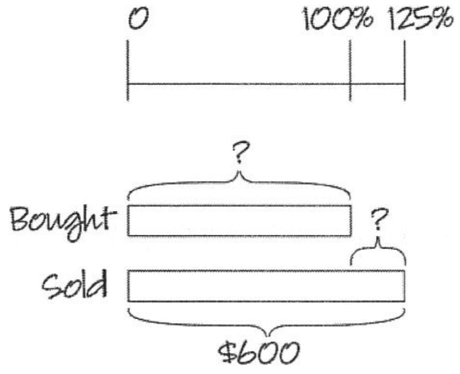
65% $\xrightarrow{③}$ \$26

100% ⟶ \$___
②
start here ①

then fill in the middle]

Ex 2 A salesman sold 2 TVs at \$600 each. The 1st was sold at a 25% profit and the 2nd was sold at a 25% loss. Find his Net profit or loss.

a) Profit on 1st TV.



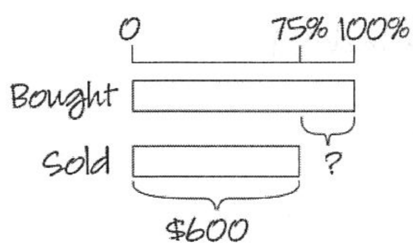
[← Scale 1st!]

	125%	\$600
÷5	25	120
x4	100%	\$480

Profit \$600 − 480 = \$120.

Profit is \$120 on 1st TV.

b) Loss on 2nd TV



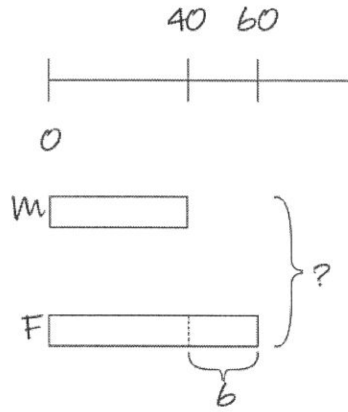
	75% →	\$600
÷3	25% →	\$200

Loss was \$200.

c) Net loss \$200 − \$120 = \$80

Moral: Equal percents (25% profit/ 25% loss) do not mean equal amounts – percents of different things!

Ex 3 [Students do] In a class, 40% of the students are male. There are 6 more female students than male. How many students are in the class?



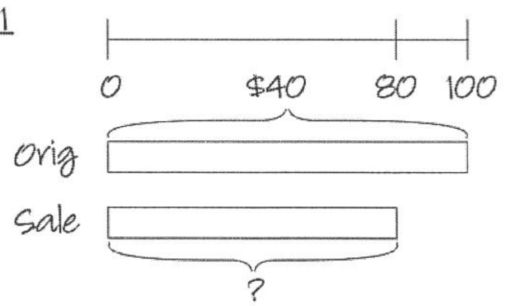
x5 20% → 6 x5

100% → ___

There are 30 altogether

Ex 4 "Multi-step" A $40 shirt was reduced 20% in a sale. Later the sales price was reduced 10%. What was the price then? [Students try first]

Step 1

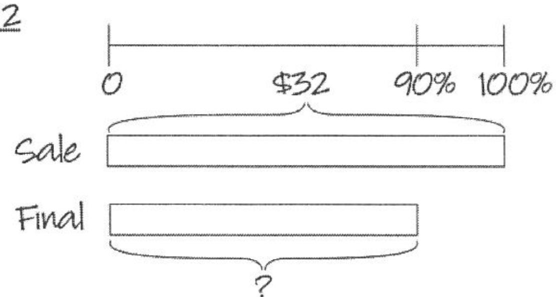


100% → 40

80% → $32

8 tenths

Step 2



100% → 32

10% → $3.20

90% → 3.2 x 9

= 32 - 3.2

= $28.80

Final cost was $28.80

Moral   Price reductions don't add!

$$\$40 \xrightarrow{-\ 20\%\text{ off}} \$32 \xrightarrow{-\ 10\%\text{ off}} \$28.80$$

$$\$40 \xrightarrow[-\ 30\%\text{ off}]{} \$28.00$$

10% off a different amount!

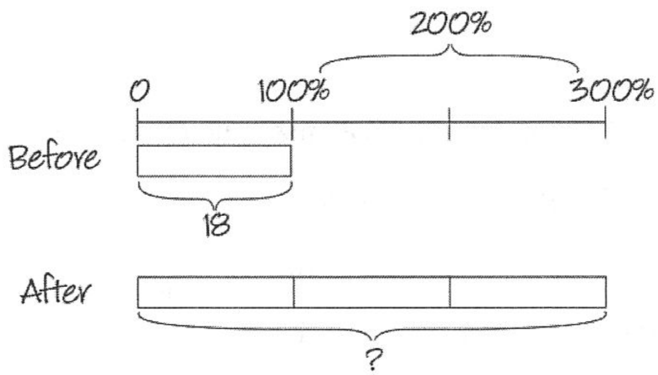
Caution on Percents

① Price of \$100 stereo was increased by 10%, then reduced by 10%

$$\$100 \xrightarrow{10\%\text{ inc.}} \$110 \xrightarrow{10\%\text{ off}} \$99$$

Not original Price!

② An \$18 Stock increased 200%



Stock rose 3 x 18 = \$54.

$\boxed{\text{Increase } 200\% \neq \text{double!}}$

In general, % problems are easy if you keep track of what 100% (or the whole amount) means.

If time: Do problems 6 (tricky), 7, 8, 9, 10 of Practice 4E.

$\boxed{\text{HW}}$ Read § 7.3 do HW set 32.

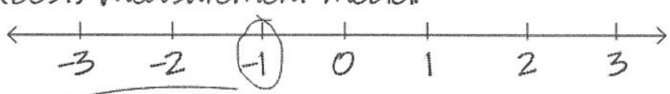
What to bring to class:
Ask students to bring PM 4A and 5A.

8.1 Integers

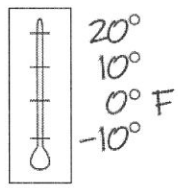
Goal: To explain $-1 \times -1 = 1$

Models

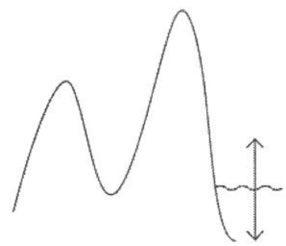
(Best) Measurement model.



Number Line



Temp



Elevation

this is a position labeled $\underbrace{-1}$ (= opposite of 1)

Single symbol, nothing to do with subtraction yet.

Def: The integers are the set of whole numbers together with their opposites.

$$(\ldots \underbrace{-3, \; -2, \; -1}_{\text{negatives}}, \; 0, \; \underbrace{1, \; 2, \; 3}_{\text{positives}}, \ldots)$$

[SAY: We are once again enlarging the set of whole numbers!]

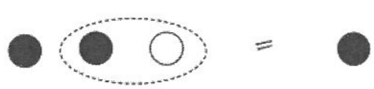
· Money $1 bill, $1 I.O.U.

[1st use I.O.U.'s, then move to chip models = abstract I.O.U.'s.]

· Set model – doesn't work well, but is used in some schools.

"arbitrary and hard to remember"

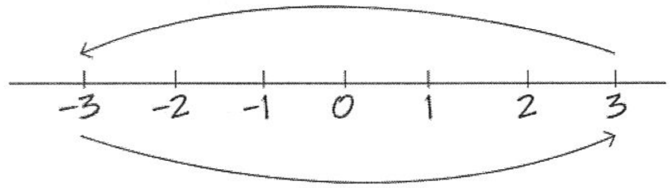
Black chip ↔ 1 Red chip ↔ −1



cancels

Operations

"Take the opposite" (A new operation!)



the opposite of 3 is -3

the opposite of -3 is 3

$-(3) = -3$
operation   number
$-(-3) = 3$

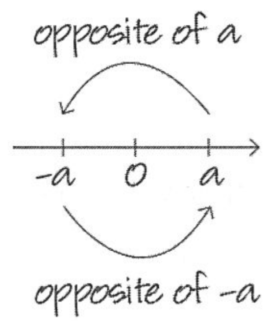
Ex a) 8 steps back is ___ steps forward.

b) A decent of 10 m means an ascent of ___.

c) $\underbrace{\text{6 steps forward}}_{6}$ is $\underbrace{\text{\_\_\_\_\_\_\_\_\_\_\_\_}}_{-(-6)}$ -6 step forward.

$6 = -(-6)$

In general



Principle: "The opposite of the opposite is the original."

↓

$\boxed{\text{Rule 1: } -(-a) = a}$ for any integer.

[SAY:

Note: · Works for both positive and negative #'s

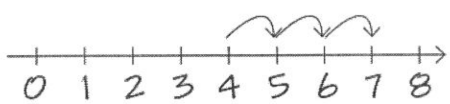
· Not the reason for $-1 \times -1 = 1$ More work!

· 3 uses for "−" : (1) opposite, label -3, subtraction]

Addition

Case 1 pos + pos

2



3

1 $4 + 3 = \underline{7}$

start ↑ count up 3 ↑

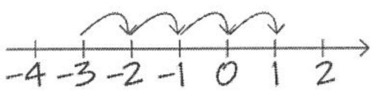
[SAY: · we had \$4 and earned \$3 more.

· the elevator started on the 4th floor and went up 3.]

Case 2 neg + pos

$-3 + 4 = \underline{1}$

start ↑ count up ↑



· the temp was $-3°$F, then rose $4°$F

· I was in debt \$3, then earned \$4 ⟵ from students

Case 3 (Hardest) pos + neg

$4 + -3$

start ↑ count up -3?? ↑

[SAY: what does this mean?

⇝ Need an interpretation!]

opposite

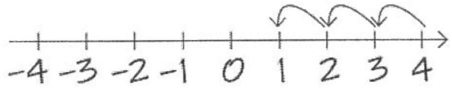
Interpretation: "count up -3" means "take the opposite of counting up 3"

$4 + -3$ = Start at 4, count up -3

= Start at 4, take the opposite of counting up 3

= " , count down 3

$= \underline{1}$



Note: ① Same as $-3 + 4 = 1 \Rightarrow$ this interpretation makes addition commutative!

② Same as subtracting! $4 + -3 = 4 - 3$.

Principle: "Adding the opposite is the same as subtracting."

$\boxed{\text{Def: } a + -b = a - b}$

· If I have a $4 in my right pocket and $3 I.O.U. in my left, How much money can I spend?

[SAY: · I was on the 5th floor and the elevator went up −2 floors.

Not realistic! We don't <u>ever</u> say that, so including problems like this makes negative #'s artificial, which they are not.

We are starting to find that integer word problems are difficult to create.]

<u>Case 4</u> neg + neg HW problem. Modify Case 3.

$\boxed{\text{HW}}$ Read § 8.1 [there is a lot of nice Teacher info]

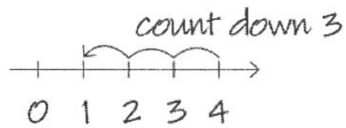
Do HW set 34. Prob 11 won't get graded, but read it.

What to bring to class:
Ask students to bring PM 4A and 5A.

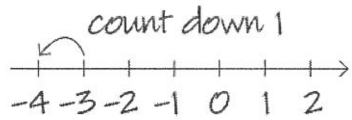
8.2 More integer basics

Subtraction

Case 1, 2 $4 - 3 = \underline{1}$



$-3 - 1 = \underline{-4}$

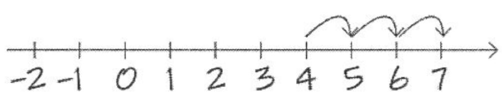


No problem.

Case 3, 4 $4 - -3$
$-2 - -3$
count down −3??

interpretation: Count down −3 = opposite of count down 3
= count up 3!

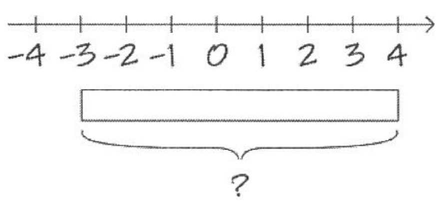
$4 - -3 = 4 + 3 = 7$
$-2 - -3 = -2 + 3 = 1$



Principle: "Subtracting a negative is the same as adding the opposite."

$\boxed{\text{Rule 2: } a - (-b) = a + b}$

Another way: $\underbrace{4 - -3}_{\text{part-whole interpretation}}$



Another way: Pattern

$4 - 2 = 2$

+1

$4 - 1 = 3$

+1

$4 - 0 = 4$

+1

$4 - (-1) = \_\_$

+1

$4 - (-2) = \_\_$

+1

$4 - (-3) = \_\_$

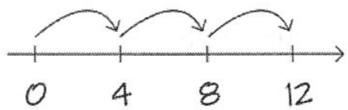
Another way: Missing addends: $4 - (-3) = \_\_ \longleftrightarrow 4 = -3 + \_\_$

Ans: $-3 + \underline{7} = 4$

[ASK!! they should be very comfortable with this by now]

Multiplication of Integers

(1) pos x pos $\quad 3 \times 4 = \underline{3 \text{ groups of } 4} = 4 + 4 + 4 = 12$



(2) pos x neg = neg

$3 \times -4 = \underline{3 \text{ groups of } -4} = -4 + -4 + -4 = -12$

(3) neg x pos = neg

$-4 \times 3 = \underbrace{-4 \text{ groups}}_{??} \text{ of } 3$ what does this mean?

Need an interpretation. Want commutative property

$-4 \times 3 = 3 \times -4 = -12$

tells us what the answer should be.

Interpretation: "the opposite of 4" groups of 3 $\leftrightarrow$ the opposite of "4 groups of 3"

so

$-4 \times 3 = -(4 \times 3) = -(12) = -12$

interpretation $\quad$ opposite operator

Thus,

$$\boxed{\text{Rule 3: } -a \times b = -(a \times b)}$$

<u>Another way:</u>

$$\begin{array}{ll} 2 \times 3 = 6 & \\ & -3 \\ 1 \times 3 = 3 & \\ & -3 \\ 0 \times 3 = 0 & \\ -1 \times 3 = \_\_ & \\ -2 \times 3 = \_\_ & \end{array}$$

<u>Case 4</u> neg x neg = pos.

Use the previous interpretation!

$$\begin{aligned} -3 \times -4 &= \text{"the opposite of 3" groups of } -4 \quad \text{Inter.} \\ &= \text{the opposite of "3 groups of } -4\text{"} \\ &= -(3 \times -4) \quad \text{Case 2} \\ &= -(-12) \quad \text{Rule 1} \\ &= 12 \end{aligned}$$

$$\boxed{\text{Hence, } -a \times -b = a \times b}$$

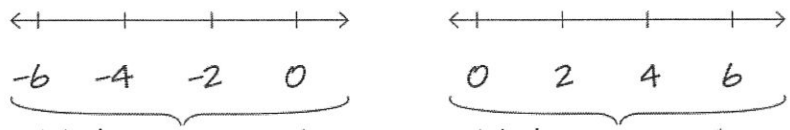
<u>Another way</u>

<u>Pattern:</u>

x	2	1	0	-1	-2
2	4	2	0	-2	-4
1	2	1	0	-1	-2
-1	-2	-1	0	1	2
-2	-4	-2	0	2	4

Division Follows from multiplication

Compare $-6 \div -2$ to $6 \div 2$ using M.D.



-6 is how many -2's 6 is how many 2's?

same question, but in
opposite direction
Ans to both: 3

$\boxed{\text{Rule 5 } -a \div -b = a \div b}$ $\boxed{\begin{array}{c}\text{-or-}\\ \frac{-a}{-b} = \frac{a}{b}\end{array}}$

Next we show $-6 \div 2$, $6 \div -2$, $-(6 \div 2)$ are equal.

$$\boxed{-(6 \div 2)} = -(6 \times \tfrac{1}{2}) = -6 \times \tfrac{1}{2} = \boxed{-6 \div 2}$$

and

$$\boxed{-6 \div 2} = -6 \div -(-2) = \boxed{6 \div -2}$$

So, $-(6 \div 2) = -6 \div 2 = 6 \div -2$

Rule 6: $-(a \div b) = -a \div b = a \div (-b)$ $\left[\text{or } \frac{a}{-b} = \frac{-a}{b} = \frac{a}{-b}\right]$

$\boxed{\text{HW}}$ Read § 8.2 [It has a lot of nice teacher help.] Do HW set 35

What to bring to class: Ask students to bring PM 4A and 5A.

8.3 Integers as numbers

Integers are numbers because they behave like them:

(1) Commutative property $-3 + 5 = 5 + -3$, $(-3)\,5 = 5\,(-3)$

(2) Assoc. Prop $(-3 + -2) + 7 = -3 + (-2 + 7)$, $-3 \times (-2 \times 7) = (-3 \times -2) \times 7$ check using Rules!

(3) Distributive Prop. $3\,(-4 + -2) = 3 \times -4 + 3 \times -2$

(4) Identities $-3 + 0 = -3$, $-5\ \ 1 = -5$.

And a new property which is important to integers:

(5) <u>Additive inverse property</u>: For each integer a there exists an integer called the opposite of a, denoted by $-a$, which satisfies

$$a + -a = 0.$$

[SAY: Additive Identity $\longleftrightarrow$ Describes 0
Mult. Identity $\longleftrightarrow$ Describes 1
Mult. Inverse $\longleftrightarrow$ Describes fractions
Additive Inverse $\longleftrightarrow$ Describes integers!]

we already have prop. for 0, 1. this is why we exclude from def of prime numbers.

<u>Summary</u>

(1) Created "opposite numbers"

(2) Made interpretations so we could +, −, x, ÷ them. Interpretation summarized by Rules 1–6.

(3) Integers satisfy properties (⇒ integers are <u>numbers</u>)

(4) Special property associated to integers.

We need to show:

(5) Properties $\Rightarrow$ Rules 1-6
"Interpretations"

(5) Means that we cannot develop a different set of interpretations, these rules are forced upon us by the properties!

Theorem: Rules 1-6 follow from the arithmetic properties.

Rule 1: $-(-a) = a$ [uses Decompress/ Analyze/ Compress proof w/ Add an appropriate 0.]

$-(-a) = 0 + -(-a)$	Additive Identity
$= (a + -a) + -(-a)$	Additive Inverse
$= a + [-a + -(-a)]$	Associative Property
$= a + 0$	Additive Inverse
$= a$	Additive Identity

Def 8.1.3 $\quad a + -b = a - b$

$a - b = \_\_$ means $a = b + \_\_$

Add $-b$ to both sides:

$a + -b = -b + (b + \_\_)$	
$= (-b + b) + \_\_$	Associative Property
$= 0 + \_\_$	Additive Inverse
$= \_\_$	Additive Identity
$= a - b$	Substitution

[so def is really a rule]

Rule 2: $a - -b = a + b$ $\quad$ Done in last HW set
[Follows from R1 and Def 8.1.3]

Rule 3: $-a \cdot b = -(a \cdot b)$

1st

Ask

$-3 \cdot 4 = -3 \cdot 4 + 0$ ___

$= -3 \cdot 4 + [3 \cdot 4 + -(3 \cdot 4)]$ ___

$= [-3 \cdot 4 + 3 \cdot 4] + -(3 \cdot 4)$ ___

$= (-3 + 3) \cdot 4 + -(3 \cdot 4)$ ___

$= 0 \cdot 4 + -(3 \cdot 4)$ ___

$= 0 + -(3 \cdot 4)$ ___

$= -(3 \cdot 4)$ ___

$= -(12) = -12$

2nd

$-a \cdot b = -a \cdot b + 0$

$= -ab + [ab + -(ab)]$

$= [-a \cdot b + ab] + -(ab)$

$= (-a + a) \cdot b + -(ab)$

$= 0 \cdot b + -(ab)$

$= 0 + -(ab)$

$= -(ab)$ ✓

Rule 4: (using Already verified Rules)

$-a \cdot -b = -(a \cdot -b)$	Rule 3
$= -(-b \cdot a)$	Commutative Property
$= -(-(ba))$	Rule 3
$= ba$	Rule 1
$= ab$	Commutative Property

Rule 5: $-a \div -b = a \div b$

what?

$-a \div -b =$ ___ means $-a = -b \cdot$ ___

Take the opposite of both sides:

$-(-a) = -(-b \cdot$ ___ $)$

$\parallel$

$a = b \cdot$ ___

$\updownarrow$

$a \div b =$ ___ $= -a \div -b$!

Def. (Ordering) $a \leq b \Leftrightarrow b - a$ is positive or zero.

Order Rules

1. $a \le b \iff a + c \le b + c$

2. $a \le b \iff \begin{cases} ac \le bc & c > 0 \\ ac \ge bc & c < 0 \end{cases}$

Proof of Order Rule 1

$a + c \le b + c \iff (b + c) - (a + c)$ is positive or zero.

$\iff b + c + -a + -c$ is pos. or 0	HW Prob
$\iff b + -a + (c + -c)$ is pos. or 0	Any order
$\iff b - a + 0$ is pos. or 0	Add inv, R2
$\iff b - a$ is pos. or 0	Add id.
$\iff a \le b.$	■

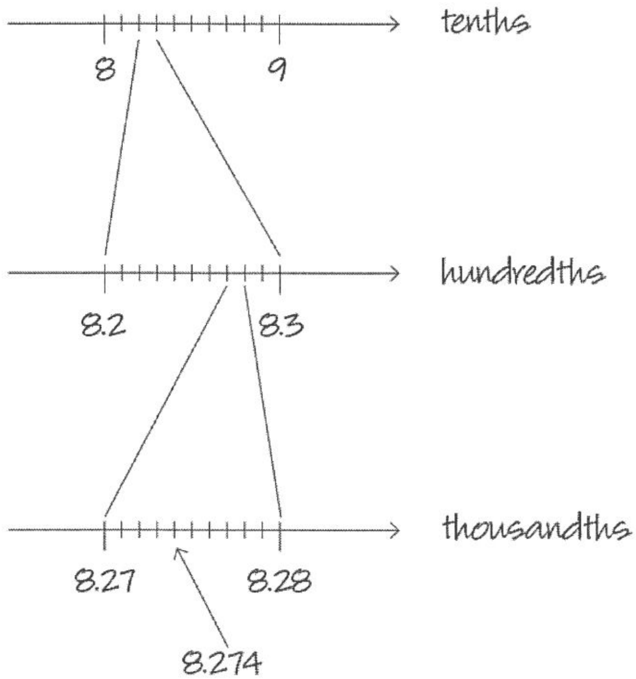
$\boxed{\text{HW}}$ Read 8.3 Do HW set 36

What to bring to class:
Ask students to bring PM 4A and 5A.

9.1 Decimals

Decimals represent points on the number line by repeatedly sudividing intervals into tenths, hundredths, etc.

<u>Ex</u> Find 8.274



this is just place value!

I. Introduction

	1's	tenths	hundredths	thousandths
Chip	○ ○ ○ ○ ○ ○ ○ ○	○ ○	○ ○ ○ ○ ○ ○ ○	○ ○ ○ ○

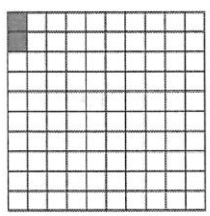
separates units from $\frac{1}{10}$'s, etc.

Expanded form:

$$8.274 = 8 \times \underbrace{1}_{\text{dollars}} + 2 \times (\underbrace{\tfrac{1}{10}}_{\text{dimes}}) + 7 \times (\underbrace{\tfrac{1}{100}}_{\text{pennies}}) + 4 \times \tfrac{1}{1000}$$

denominations!

Taught by:
- number line (meter sticks, balance scales. . .)
- "Hundreds square" used <u>but</u>
  - (i) 2 dimensional
  - (ii) kids make error:

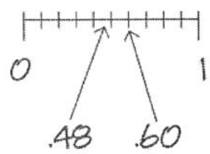
    .2 = 
  - (iii) what about thousandths?
- Chip models

Notation:

$$\underbrace{3.14}_{\text{U.S.}} \qquad \underbrace{3{,}14}_{\text{some European Countries}} \qquad \underbrace{3\cdot 14}_{\text{Singapore}}$$

Decimals are easy to compare by: .48___ .6

· Locating on number line



· Making "equal lengths"

.60

.48

· Convert to like fractions

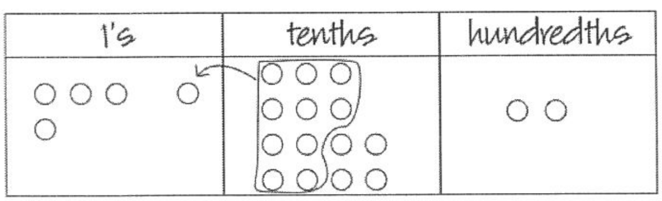
$.6 = \frac{6}{10} = \frac{60}{100}$ $\quad .48 = \frac{48}{100}$

Operations – Same as for whole numbers but keep track of the decimal point.

II. Addition & Subtraction

Ex 1

$$\begin{array}{r} {\scriptstyle 1} \\ 3.62 \\ +\ 1.8 \\ \hline 5.42 \end{array}$$



Ex 2 [Students do 1 min.]

$$\begin{array}{r} 11.1700 \\ \underline{-2.8613} \end{array}$$

Chip Model:

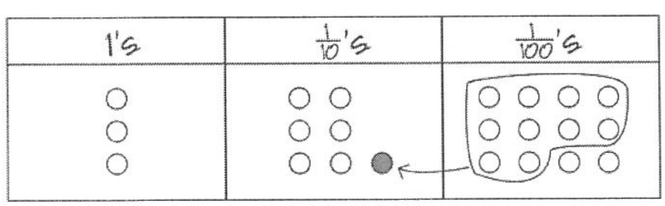
· Allign Place Values!

· Append 0's until same length

III. 1-digit Multiplication [Students do, 1 min]

Ex 3

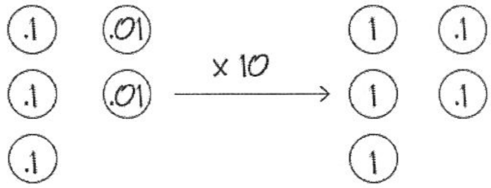
$$\begin{array}{r} {\scriptstyle 1} \\ 1.24 \\ \times\quad 3 \\ \hline 3.72 \end{array}$$



IV. Place Value

Principle: "multiplying by 10 means shift the decimal point to the right."

Why?



$$.32 \times 10 = 3.2$$

$$.367 \underset{\div 10}{\overset{\times 10}{\rightleftarrows}} 3.67 \quad \text{because} \quad \frac{367}{1000} \underset{\div 10}{\overset{\times 10}{\rightleftarrows}} \frac{367}{100}$$

$$.367 \underset{\div 100}{\overset{\times 100}{\rightleftarrows}} 36.7 \quad \text{because} \quad \frac{367}{1000} \underset{\div 100}{\overset{\times 100}{\rightleftarrows}} \frac{367}{10}$$

Hence: multiply by 10, 100, 1000, . . . $\longleftrightarrow$ move decimal 1, 2, 3, . . . places to the right

divide by 10, 100, 1000, . . . $\longleftrightarrow$ move decimal 1, 2, 3, . . . place to the left

V. Multi digit Multiplication & Division

1st reg. algorithm!

Ex 4 $\quad 1.02 \times 3.4 = \underset{2}{\frac{102}{100}} \times \underset{1}{\frac{34}{10}} = \underset{3}{\frac{102 \times 34}{1000}} = \frac{3468}{1000}$

$= 3.468$

2nd Shift decimal pt!

$$\begin{array}{r} 1.02 \\ \times\ 3.4 \\ \hline 408 \\ 3060 \\ \hline 3.468 \end{array}$$

1st Do reg. algorithm

2nd Shift decimal

Ex 5 [Students do]

$$\begin{array}{r} .02 \\ \times\ .041 \\ \hline \end{array}$$

For division, use compensation method from mental math!

$$\underset{}{\overset{\times 10}{162.8}} \div \overset{\times 10}{.037} = \overset{\times 10}{1628} \div \overset{\times 10}{.37}$$

$$= \overset{\times 10}{16280} \div \overset{\times 10}{3.7}$$

$$= 162800 \div 37 \longleftarrow \text{whole \# division!}$$

– or –

$$162.800 \div .037 = 162800 \div 37$$

(2) shift 3 (1) shift 3

We can do this because of equivalence of fractions:

$$162.8 \div .037 = \frac{1628 \times 10}{.037 \times 10} = \frac{1628 \times 10}{.37 \times 10} = \frac{16280 \times 10}{3.7 \times 10}$$

$$= \frac{162800}{37} = 162800 \div 37$$

Ex 6 [Students do] Find the value of $.81 \div 3.9$ to 2 decimal places.

$$3.9\overline{)\,.81} \longrightarrow \begin{array}{r} 6 \quad .207 \\ 39\overline{)\,8.1} \\ -7\,8 \\ \hline .30 \\ -0 \\ \hline 300 \\ -273 \\ \hline 27 \end{array}$$

$$\boxed{.81 \div 3.9 \approx .21}$$

$\boxed{\text{HW}}$ Read section 9.1 Do HW set 37

What to bring to class:
Ask students to bring PM 4A and 5A.

9.2 Fractions and Decimals [$1\frac{1}{2}$ days]

I. Converting Decimals to Fractions.

Denominator = appropriate power of 10

$.37 = \frac{37}{100}$

$3.288 = \frac{3288 \div 8}{1000 \div 8} = \frac{411}{125}$

$1000 = 2^3 \cdot 5^3$

Denominator = Product of 2's and 5's

Numerator = When no 2's or 5's, fraction in simplest form.

II. Fraction ⟶ decimal.

(a) Use equivalent fractions until denominator is a power of 10.

$\frac{71}{100} = .71$

$\frac{13}{25} = \frac{52}{100} = .52$

Student does ⟶ $\frac{3}{8} = \frac{3 \cdot 5 \cdot 5 \cdot 5}{2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5} = \frac{375}{1000} = .375$

⟶ $\frac{7}{20} =$

or (b) Just divide

$$\frac{1}{8} = 8\,\overline{)\,1.00}\quad \begin{array}{r} .125 \\ 1.00 \\ \underline{8} \\ .20 \\ \underline{16} \\ 40 \end{array}$$

$\frac{1}{3} = .3333\ldots\ldots$

Two types of decimal numbers:

.382 terminating (ends)

.333 ... nonterminating (no end)

Must always check!

<u>Thm</u> A fraction $\frac{a}{b}$ in $\boxed{\text{simplest form}}$ has a terminating decimal expansion $\Longleftrightarrow$ denominator b is a product of 2's and 5's only.

<u>Ex</u> $\frac{2}{25}$ $\quad \frac{6}{150}$ $\quad \frac{5}{321}$ $\quad \frac{21}{35}$

<u>Sketch of Proof</u>: One way "$\Longrightarrow$" A fraction with a terminating decimal expansion can be written $\frac{a}{b} = \frac{\text{whole \#}}{\text{power of 10}} = \frac{\text{whole \#}}{10^n} = \frac{\text{whole \#}}{2^n \cdot 5^n}$

<u>Ex</u> $.882 = \frac{882}{10^3} = \frac{882}{2^3 \cdot 5^3} = \frac{441}{2^2 \cdot 5^3}$

Power of 10

there may be cancelation, but denominator still product of 2's, 5's.

<u>Other way</u>: If fraction has form say $\frac{N}{2^3 \cdot 5^7}$ multiply by $\frac{2}{2}$ or $\frac{5}{5}$ until powers of 2 and 5 match in denominator.

$$\frac{N}{2^3 \cdot 5^7} = \frac{2}{2} \cdot \frac{2}{2} \cdot \frac{2}{2} \cdot \frac{2}{2} \cdot \frac{N}{2^3 \cdot 5^7} = \frac{2^4 \cdot N}{2^4 \cdot 2^3 \cdot 5^7} = \frac{16N}{2^7 \cdot 5^7}$$

$$= \frac{16N}{10^7} \longleftarrow \text{terminating decimal}$$

If denominator has factors other than 2's or 5's, what happens?

<u>Ex</u>

$\frac{8}{37} = .216\ 216\ 216\ldots$

$= .\overline{216}$

Period 3

Repeating digits called the <u>repetend</u>.

$$
\begin{array}{r}
.216216216 \\
37\,\overline{)\,8.000} \\
\underline{-74} \\
60 \\
\underline{37} \\
230 \\
\underline{-222} \\
80 \\
\underline{-74} \\
60 \\
\underline{-37} \\
230 \\
\underline{-222} \\
8
\end{array}
$$

pattern repeats forever

repeats

repeats

<u>Ex</u> [Students do]

$\frac{9}{22} = .4\overline{09}$

period 2

$$
\begin{array}{r}
.409090 \\
22\,\overline{)\,9.0} \\
\underline{88} \\
200 \\
\underline{198} \\
200 \\
\underline{-198} \\
20
\end{array}
$$

repeat

repeat

<u>Ex</u> $\frac{1}{7} = .\overline{142857}$

Split amongst class:

$\frac{2}{7} = .\overline{285714}$

$\frac{3}{7} =$

$\frac{4}{7} =$

$\frac{5}{7} =$

$\frac{6}{7} =$

Ask: the period in each case is $\leq$?

$$
\begin{array}{r}
.142857 \\
7\,\overline{)\,1.0} \\
\underline{7} \\
30 \\
\underline{-28} \\
20 \\
\underline{14} \\
60 \\
\underline{-56} \\
40 \\
\underline{-35} \\
50 \\
\underline{-49} \\
1
\end{array}
$$

repeat

Basic Fact: Every fraction can be represented as a repeating decimal.

Proof: To convert $\frac{a}{b}$ to a decimal, we find $a \div b$ or $b\overline{)a}$. At each step in the long division the remainder r is a whole # with $0 \leq r < $ denominator.

[Explain by circling remainders in $\frac{1}{7}$ case.]

$\Longrightarrow$ only b possibilities for r

$\Longrightarrow$ repeats after at most b steps.

$\Longrightarrow$ once a remainder repeats, long division steps must repeat. $\square$

2 Cases:

- If some remainder $r = 0$, then it is a terminating decimal (still regard as repeating $\frac{1}{2} = .5 = .50000\ldots = .5\overline{0}$)
- Otherwise repeats with period $\leq b - 1$

Ex [If time, do calc. If not, write answer]

$\frac{1}{17}$

[Make table before you start!]

17	1
34	2
51	3
68	4
85	5
102	6
119	7
136	8
153	9

$17\overline{)1.000000}$ = .0588235294117647

- -85
- 150
- -136
- 140
- -136
- 40
- -34
- 60
- -51
- 90
- -85
- 50
- -34
- 160
- -153
- 70
- -68
- 20
- -17
- 30
- -17
- 130
- -119
- 110
- -102
- 80
- -68
- 120
- -119
- 1

One can't see $\frac{1}{17}$ repeats on a calculator! This important concept can only be understood by students who know long division.

Ex $\quad \frac{1}{13} = .\overline{076923}$

period is 6 (not 12)

Other way: Repeating decimals $\longrightarrow$ fractions!

Ex $\quad$ Write $.\overline{17}$ as a fraction

set $x = .\overline{17}$

constant

$$\begin{aligned} 100x &= 17.171717\ldots \\ -x &= -.171717\ldots \\ \hline 99x &= 17 \\ x &= \tfrac{17}{99} \longrightarrow \boxed{.\overline{17} = \tfrac{17}{99}} \end{aligned}$$

Ex $\quad .\overline{361} \qquad x = .\overline{361}$

students try

$$\begin{aligned} 1000x &= 361.361361\ldots \\ -x &= -.361361\ldots \\ \hline 999x &= 361 \longleftarrow \text{whole \#!} \\ x &= \tfrac{361}{999} \longrightarrow \boxed{.\overline{361} = \tfrac{361}{999}} \end{aligned}$$

[Caution: "rule" $\frac{a}{999}$ only true if there are no initial non repeating digits.]

Ex $\quad .11\overline{32} = \frac{1121}{9900}$

$$\begin{aligned} 10000x &= 1132.3232\ldots \\ -100x &= -11.3232\ldots \\ \hline 9900x &= 1121 \\ x &= \tfrac{1121}{9900} \end{aligned}$$

$$\left.\begin{array}{l} \underline{\text{Ex}} \quad .2\overline{31} \\ 5.\overline{173} \\ 1.2\overline{34} \\ .9 \end{array}\right\} \text{Students do (especially)}$$

<u>Fraction – Decimal theorem</u>

(a) Every fraction can be written as a repeating decimal and <u>vice versa.</u>

(b) the decimal form terminates $\Longleftrightarrow$ in simplest form the denominator is a product of 2's and 5's only.

$\longrightarrow$ otherwise repeats of period $\leq$ (denominator -1)

Note: Terminating decimals have 2 repeating forms

$$1 = \begin{cases} 1.0000\ldots \\ .9999\ldots \end{cases} \qquad \frac{1}{4} = .25 = \begin{cases} .250000\ldots \\ .249999\ldots \end{cases}$$

[Fraction $\longleftrightarrow$ decimal correspondence has no other ambiguity]

$\boxed{\text{HW}}$ Read § 9.2 do HW set 37

What to bring to class:
Ask students to bring PM 4A and 5A.

9.3 Rational and Real #'s [$1\frac{1}{2}$ days]

(1) Divisions like $5 \div 3$ did not have whole # solutions

$\Longrightarrow$ Enlarged whole #'s to get fractions:

$$5 \div 3 = \frac{5}{3}$$

New Property: Multiplicative Inverse: $x \cdot \frac{1}{x} = 1$

(2) Subtraction like $2 - 8$ did not have a solution in whole #'s or fractions.

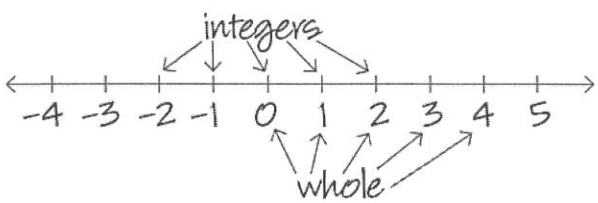
$\Longrightarrow$ Enlarge whole numbers to get integers.

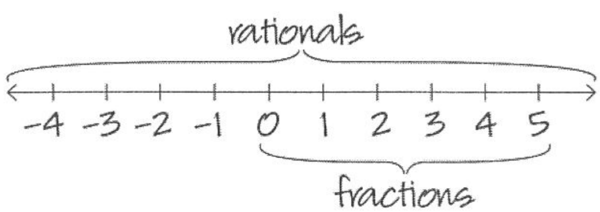
New Property: Additive inverse: $a + -a = 0$.

Doing both gives

<u>Def</u> : the <u>rationals</u> are the set of fractions together with their opposites.

<u>Ex</u> $\frac{-3}{8}$ is a rational, but not a fraction or integer.





Rationals satisfy the complete list of Arithmetic Prop.

- Commutative Property $\quad a + b = b + a,\ ab = ba$
- Associative Property $\quad a + (b + c) = (a + b) + c \qquad a(bc) = (ab)c$
- Distributive Property $\quad a(b + c) = ab + ac$
- Additive and Multiplicative Identity $\quad \underbrace{a + 0 = a,}_{\text{defines } 0} \quad \underbrace{a \cdot 1 = a}_{\text{defines } 1}$
- Additive and Multiplicative Inverses $\quad \underbrace{a + -a = 0}_{\text{integers}} \quad \underbrace{a \cdot \frac{1}{a} = 1}_{\text{fractions}}$

New!
- Closure: $a + b$, $a - b$, $a \times b$, $a \div b$ are all rational numbers

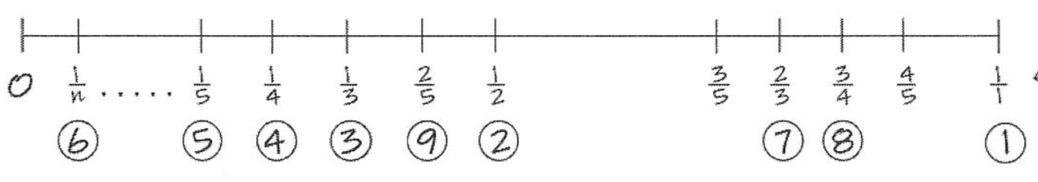
Note:
- Complete list of Arithmetic properties – Every statement, identity, rule, etc. follows from them.
- Used closure property throughout the course, but didn't make it explicit.

Density: How many fractions in the interval $[0, 1]$?

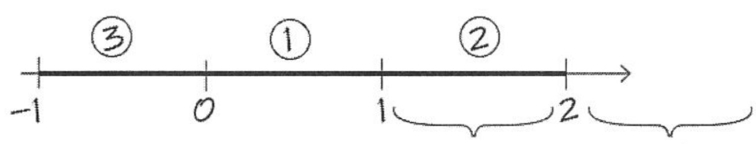
At least as many whole #'s: $0, \frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5} \ldots \frac{1}{n}$

More: $\frac{2}{3}, \frac{3}{4}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5} \ldots$

Fill-in in order ①, ②, ③ . . .



We could continue filling in pts forever



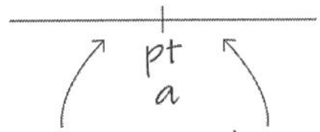
④ Can do the same for $[-1, 0]$ interval $[1, 2]$, $[2, 3]$ etc . . .

Leads to 2 questions:

Q1 Is every pt on the number line a rational #?

[Answer Later]

Q2 Given any pt on "   ", are there rational numbers "near by?"



Is there a rational # near a?

<u>Answer to Q2:</u> Yes, because rational #'s are <u>dense</u>, i.e. any interval contains at least one rational #.

To see why,

Fix a point a and look at intervals

$[a, a+\frac{1}{2}]$

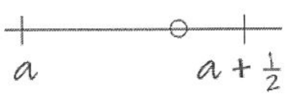
$[a, a+\frac{1}{3}]$

$[a, a+\frac{1}{4}]$

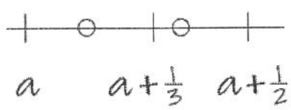
$[a, a+\frac{1}{n}]$

<u>rational in interval?</u>

density $\Longrightarrow$ yes!

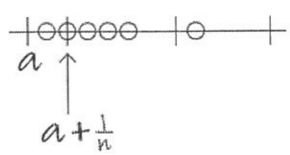


density $\Longrightarrow$ yes!



In fact, any interval contains an ∞ # of rationals!

density $\Longrightarrow$ yes!



How to find a rational in an interval:

Ex Find a rational # between 3.141 and π

Pick any:

$3.14100\ldots$ — 3.1411

$3.14159\ldots$ — 3.1412 ✓

3.1413

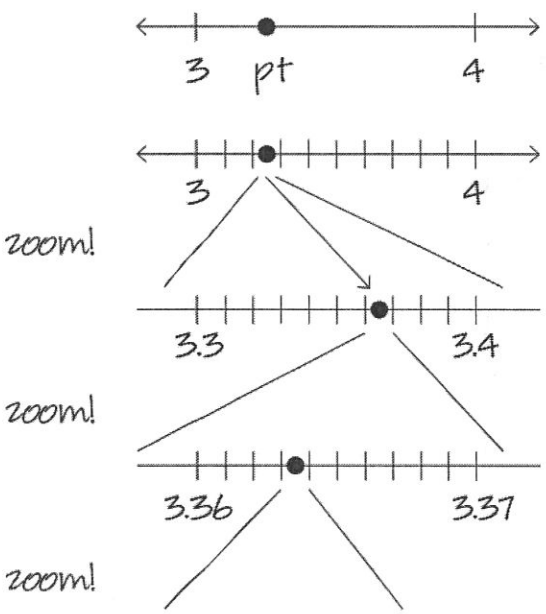
3.1414

$3.1412 = \frac{31412}{10000} \longleftarrow$ rational!

Real numbers

To Answer Q 1, we need to see that every pt on the #-line corresponds to a number.

Algorithm for converting pts – to a number



What # does this pt represent?

$3.3 \le pt \le 3.4$

$3.36 \le pt \le 3.37$

$3.364 \le pt \le 3.365$

Do this forever!

$\boxed{pt = 3.3643596 82\ldots}$

Called an infinite decimal expansion.

Note: Real numbers also satisfy the complete list of Arith. Prop.

Ex (a) $13 = 13.000\ldots$

$-.15 = -.15000\ldots$

$\frac{1}{3} = .3333\ldots$

$.\overline{183} = .183183183\ldots$

<u>Correspondence Thm</u>

repeating decimals $\longleftrightarrow$ rationals!
($\pm$ fractions)

Every rational number is also a real number.

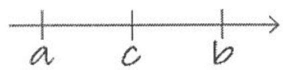
(b) $.101001000100001\ldots$ No repetend!

By correspondence Thm. this is <u>not</u> rational. Called an <u>irrational number</u> because it can't be written as $\frac{a}{b}$ for any integers a, b.

Answer to Q1: NO!

<u>Thm</u> Irrational number are also dense.

<u>Proof</u>: Given an interval $[a,b]$ find a finite decimal c in the interval.



$$\underbrace{.137\ldots6}_{\text{decimal \#c}}\ \underbrace{0000\ldots0}_{\text{enough 0's to stay in interval}}\ \underbrace{12112111211112}_{\text{any non repeating sequence.}}\ldots$$

$\square$

Ex [HW Prob] Find it irrational between $.\overline{67}$ and $.\overline{68}$

$.67676767\ldots$

$.68686868\ldots$

$.\overline{67} < .68\underline{0} < .\overline{68}$

$\uparrow$

rational

Irrational: $.680000123456789101112\ldots$

Conclusions:

- there are 2 types of real numbers: rational and irrational.
- Both rationals and irrationals are dense (infinitely many in any interval).

$\boxed{\text{HW}}$ Read § 9.3 do HW set 39

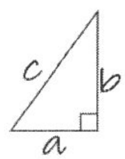
What to bring to class:
Ask students to bring PM 4A and 5A.

9.4 Newtons Method and $\sqrt{2}$

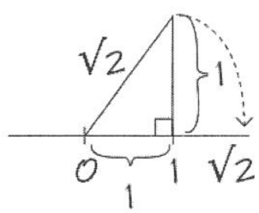
[SAY: We've learned: not all numbers are rational! So far, looks like irrational #'s are just theoretical since we can't even write one down (infinitely many digits)
In fact, any building contractor will tell you the opposite – irrational #'s occur naturally and are used frequently.]

Square roots – are often irrational. [SAY: To understand, we'll look at $\sqrt{2}$ in detail.]

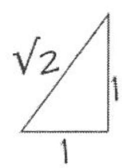
1. Finding $\sqrt{2}$: Pythagorean Thm:



$a^2 + b^2 = c^2$



$\longrightarrow$



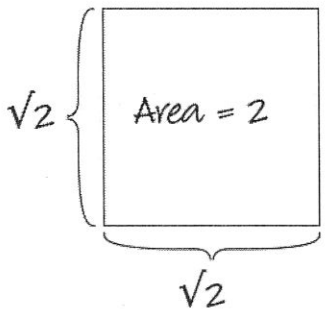
$1^2 + 1^2 = (\sqrt{2})^2$

$\sqrt{2}$ is a pt on the # line $\Longrightarrow$ its a real number.

II. Find an infinite decimal expansion

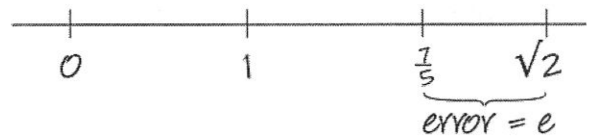
Construct a square of Area = 2

[ASK: How long is each side?]



Approximate $\sqrt{2} \approx \frac{7}{5} = 1.4$

$(1.4^2 = 1.96 < 2)$



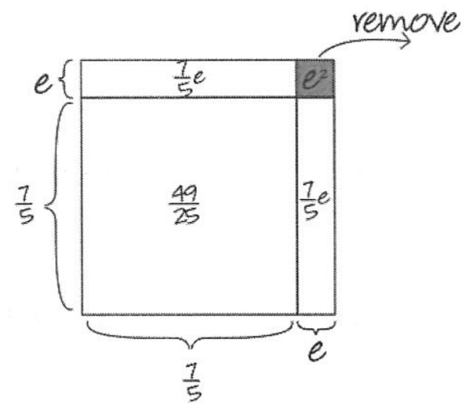
$\sqrt{2} = \frac{7}{5} + e$

$2 = (\frac{7}{5} + e)^2 = \frac{49}{25} + 2 \cdot \frac{7}{5} \cdot e + e^2$ remove

$2 \approx \frac{49}{25} + \frac{14}{5} e$ $\quad$ ×25

Solve for e $\Longrightarrow 50 \approx 49 + 70e$

$\boxed{e \approx \frac{1}{70}}$



Hence a better estimate for $\sqrt{2}$

$\sqrt{2} = \frac{7}{5} + e \approx \frac{7}{5} + \frac{1}{70} = \frac{99}{70} = 1.414$

A better estimate!

Algebraically: Approximate by a (instead of $\frac{7}{5}$)

$\sqrt{2} = a + e \Longrightarrow 2 = a^2 + 2ae + e^2$ remove

solve for e $\Longrightarrow 2 - a^2 \approx 2ae \Longrightarrow \frac{2-a^2}{2a} \approx e$

$\Longrightarrow e \approx \frac{1}{a} - \frac{a}{2}$

New Approximation:

$a_{new} = a + e = a + \frac{1}{a} - \frac{a}{2} = \frac{a}{2} + \frac{1}{a}$

Applying repeatedly gives:

Approx:

$$1 \longrightarrow \tfrac{1}{2} + 1 = \tfrac{3}{2} = 1.5$$

$$1.5 \longrightarrow \tfrac{3}{4} + \tfrac{2}{3} = \tfrac{17}{12} = 1.41\overline{6}$$

$$\tfrac{17}{12} \longrightarrow \tfrac{17}{24} + \tfrac{12}{17} = \tfrac{577}{408} = 1.41\overline{4215686274509803 9}$$

open book

better and better approx.

(calc read)

Note: ① $\sqrt{2}$ is not 1.414213562 followed by ∞ # of random digits.

(as many students believe)

② In each step there is always error > 0. Hence it appears that $\sqrt{2}$ is not rational. (if $e = 0$, then we would get $\sqrt{2}$ rat.)

Fact: $n = P_1 P_2 P_3 \ldots P_k$ is a prime factorization. then $n^2 = (P_1 \ldots P_k)(P_1 \ldots P_k)$ has an even # (2k's worth) of primes in its P.F.

Thm $\sqrt{2}$ is irrational.

Proof: Suppose $\sqrt{2}$ is rational. then

$$\sqrt{2} = \tfrac{a}{b} \text{ for some } a, b \text{ whole \#'s.}$$

Square both sides: $2 = \frac{a^2}{b^2}$

$$\Longrightarrow \underbrace{a^2}_{\substack{\text{even \#}\\ \text{of primes}\\ \text{in P.F.}}} = \underbrace{\underbrace{2b^2}_{\substack{\text{even \# +1}\\ \text{number of primes}\\ \text{in P.F.}}}}_{\text{odd \# of primes!}}$$

the same number can't have both an even number and odd # of primes in its P.F.

Contradictions. $\sqrt{2}$ is not rational $\Longrightarrow$ $\sqrt{2}$ is irrational.

thm If a whole # n is not a square ($n \neq 1, 4, 9, 16, \ldots$)

then $\sqrt{n}$ is irrational

$\boxed{\text{HW}}$ Read § 9.4 Do HW set 40

Radical Rules:

(1) $\sqrt[n]{a^m} = (\sqrt[n]{a^m})^m$

(2) $\sqrt[n]{a}\,\sqrt[n]{b} = \sqrt[n]{ab}$

Proof of 2: $\sqrt[n]{a} = x, \sqrt[n]{b} = y \Longleftrightarrow x^n = a, y^n = b$

then $\qquad ab = x^n y^n = (xy)^n$

PR4

By def: $\qquad \sqrt[n]{ab} = xy = \sqrt[n]{a}\,\sqrt[n]{b} \qquad \blacksquare$

[SAY: Radical Rule 2 follow from PR4 – Can we write $\sqrt[n]{a}$ as an exponent?]

Suppose we could write $\sqrt{3} = 3^{\square}$ some exponent

what is $\square$?

We know $(\sqrt{3})^2 = 3^1 \Longrightarrow 3^1 = (3^{\square})^2 = 3^{2\cdot\square}$

Equating we get $2 \cdot \square = 1 \Longrightarrow \square = \frac{1}{2}$!

$\sqrt{3} = 3^{\frac{1}{2}} \longleftarrow$ fraction exponents!

Def: Let a be any non negative real number and n a positive integer; then

$$a^{\frac{1}{n}} = \sqrt[n]{a}.$$

Rule 1

$$\sqrt[n]{a^m} = (a^m)^{\frac{1}{n}} = a^{\frac{m}{n}} = (a^{\frac{1}{n}})^m = (\sqrt[n]{a})^m$$

Rule 2

$$\sqrt[n]{a}\,\sqrt[n]{b} = a^{\frac{1}{n}} b^{\frac{1}{n}} = (ab)^{\frac{1}{n}} = \sqrt[n]{ab}$$

Ex

$$36^{3/2} = (\sqrt{36})^3 = 6^3 = 216$$

$$1024^{6/20} = 1024^{3/10} = (\sqrt[10]{1024})^3 = 2^3 = 8$$

$\boxed{\text{HW}}$ Read § 9.4 Do HW set 39

(Don't do 1 or 9).

What to bring to class:
Ask students to bring PM 4A and 5A.

Summary of the book

From now on, we will work with the real numbers.

<u>The properties of equality (or what "is" is)</u>

1) Equality of <u>reflexive</u>: $a = a$

2) Equality of <u>symmetric</u>: $a = b \Longrightarrow b = a$

$$a = 3 \Longrightarrow 3 = a$$

3) Equality of transitive: If $a = b$ and $b = c \Longrightarrow a = c$

4) Property of substitution: Any quantity may be substituted for an equal quantity in any mathematical statement without changing the truth or falsity of the statement.

$$4) \Longrightarrow \begin{cases} 5) \text{ If } a = b \Longrightarrow a + c = b + c \\ 6) \text{ if } a = b \Longrightarrow ac = bc \end{cases}$$

Equality ("=") is used to create identities.

$a = b$ means a and b stand for the same number.

The most basic identities are the arithmetic

Properties (Commutative Property, Associative Property, Distributive Property, Additive Identity, etc. . .)

From the Arithmetic Properties we can derive the most fundamental identities (rules)

Power Rules

1. $a^m a^n = a^{m+n}$
2. $a^m \div a^n = a^{m-n}$
3. $(a^m)^n = a^{mn}$
4. $a^m b^m = (ab)^m$

written in a different form

Radical Rules

1. $\sqrt[n]{a^m} = (\sqrt[n]{a})^m$
2. $\sqrt[n]{a}\,\sqrt[n]{b} = \sqrt[n]{ab}$

def

1. $a^n = \underbrace{a \cdot a \ldots a}_{n}$

notation

2. $\sqrt[n]{a} = a^{\frac{1}{n}}$

notation for

Fraction Rules

1. $\frac{an}{bn} = \frac{a}{b}$
2. $\frac{a}{b} \pm \frac{c}{b} = \frac{a \pm c}{b} \Longrightarrow \frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$
3. $a \div b = \frac{a}{b}$
4. $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$
5. $\frac{a}{b} \div \frac{c}{b} = a \div c \Longrightarrow \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$

def $\quad \frac{a}{b} = a \cdot \frac{1}{b}$

notation for

Integer Rules

1. $-(-a) = a$
2. $a - -b = a + b$
3. $-axb = -(axb)$
4. $-ax-b = axb$
5. $-a \div -b = a \div b$
6. $-a \div b = a \div -b = -(a \div b)$

def

$a - b = a + (-b)$

notation for

Order Rules

1. $a \leq b \Longleftrightarrow a + c \leq b + c$

2. $a \leq b \Longleftrightarrow \begin{cases} ac \leq bc & c < 0 \\ ac \geq bc & c < 0 \end{cases}$

def

$a < b \Longleftrightarrow b - a$ is positive

there are a couple of other important Rules

1. $a \cdot 0 = 0$ for all a (you actually need to prove $0 + 0 = 0$ First!)
2. If $ab = 0$ then either $a = 0$ or $b = 0$
3. If $a + c = b + c$, then $a = b$
4. If $ac = bc$ and $c \neq 0$, then $a = b$

Proof of 1 (assuming $0 + 0 = 0$)

$a(0) = a \cdot 0 + 0$ <u>why?</u>

$= a \cdot 0 + [a \cdot 0 + -(a \cdot 0)]$

$= [a \cdot 0 + a \cdot 0] + -(a \cdot 0)$

$= a(0 + 0) + -(a \cdot 0)$

$= a \cdot 0 + -(a \cdot 0)$

$= 0$

Ad nauseum

Proof of 2

If $a = 0$ the theorem is proved. If $a \neq 0$, then $\frac{1}{a}$ exist by mult. inverse.

$\frac{1}{a}(a \cdot b) = 0 \cdot \frac{1}{a}$ property 6 of "="

Associative Property | (1) above

$(\frac{1}{a} \cdot a)b = 0$

Mult. inverse

$1 \cdot b = 0$

Mult. identity

$b = 0$

Either way, $a = 0$ or $b = 0$

Proof of 3

$a + c = b + c$	
$(a + c) + -c = (b + c) + -c$	Property 6 of "="
$a + (c + -c) = b + (c + -c)$	Associative Property
$a + 0 = b + 0$	Additive Inverse
$a = b$	Additive Identity.

$\blacksquare$

Proof of 4

If $ac = bc \Longrightarrow ac - bc = 0$

$(a - b)\, c = 0$

by (2) either $a - b = 0$ or $c = 0$

since $c \neq 0 \Longrightarrow a - b = 0$ or $a = b$ $\blacksquare$

And thus, algebra begins:

- Equations, fractions, graphs
- Exponential functions
- Trigonometric functions
- Polynomials (and complex #'s)
- Logarithmic functions
- etc.

Where does one start?

<u>The problem</u>: Algebra by itself is like having a powerful tool with nothing to use the tool on.

Geometry provides the problems which makes algebra useful and interesting in grade school.

Onward to Geometry!