How NCTQ evaluates textbooks used in elementary mathematics courses

Because the second indicator of **Standard 5: Common Core Elementary Mathematics** requires that textbooks used in elementary math courses support instruction on essential topics of elementary math, only textbooks designed to provide teacher audiences with appropriate elementary math content are considered relevant for evaluation of coursework under the standard.[1]

After a required textbook is determined to be relevant, it is reviewed by one of the <u>mathematicians</u> who served on the Mathematics Advisory Group formed for NCTQ's national study of the preparation in math of elementary teachers (*No Common Denominator*, 2008). The mathematician does a comprehensive analysis of the textbook to determine the adequacy of its treatment of the 12 topics identified as essential by the advisory group:

- Whole numbers and place value
- Fractions and integers
- Decimals (including ratio, proportion, percent)
- Estimation
- Constants, variables, expressions, equations
- Graphs and functions
- Measurement
- Basic concepts in plane and solid geometry
- Polygons and circles
- Perimeter, area, surface area
- Volume
- Probability and data display and analysis

Topics are assessed on the basis of:

- Coverage
- Connection
- Integrity
- The sufficiency and significance of examples
- Whether the text addresses methods of teaching

---

1 Required content textbooks in elementary math content courses that do not address elementary math content or are not designed for a teacher audience cannot provide any instruction on appropriate connections to the elementary classroom. Required methods textbooks in elementary math content courses do not deliver sufficient content.

The mathematicians on the Mathematics Advisory Group consider word problems of paramount importance in elementary math content coursework and pays particular attention in their reviews to the sufficiency and appropriateness of word problems.

Separate scores are produced on four subject areas: numbers and operations, algebra, geometry and data analysis.

New editions of textbooks previously evaluated are reviewed to determine if the material in the new edition remedies any deficiencies noted in earlier evaluations. If so, the textbook's score in the relevant subject area(s) is increased. If not, the textbook's scores from the previous edition are simply carried forward to the new edition.

The scores of required primary elementary math content textbooks found in required coursework are listed below. Following the scores is a list of non-elementary math content textbooks that are the required primary textbooks found in required elementary math content coursework (or coursework described as required for elementary teacher candidates and designed for such candidates). The latter textbooks were not reviewed and their scores are given as "zero" in the scoring algorithm for course evaluation under the standard.[2]

---

[2] The only exception is data analysis texts used in data analysis courses designed for teacher candidates. These texts receive a score of 19 points out of 19 available points without any review. The mathematicians that evaluate textbooks determined that textbook coverage of the other subjects was of more importance and deserved focus, and they saw little to discriminate among textbooks or parts of textbooks dealing with data analysis.

Elementary mathematics content textbook scores

Primary author	Textbook	Numbers & Operations (54 points possible)	Algebra (39 points possible)	Geometry (54 points possible)	Data Analysis & Probability (19 points possible)	Total Score (166 points possible)
Bassarear	*Mathematics for Elementary School Teachers (5th ed.)*	21	3	33	19	76
Beckmann	*Mathematics for Elementary Teachers with Activities (4th ed.)*	54	29	48	19	150
Bennett	*Mathematics for Elementary Teachers: A Conceptual Approach (9th ed.)*	33	15	41	19	108
Billstein	*A Problem Solving Approach to Mathematics for Elementary School Teachers (11th ed.)*	35	33	50	19	137
Burris	*Understanding the Math You Teach: Content and Methods for Prekindergarten Through Grade 4*	14	0	19	0	33
Fierro	*Mathematics for Elementary School Teachers*	51	34	52	19	156
Jones	*A Mathematical Foundation for Elementary Teachers*	44	0	42	19	105
Long	*Mathematical Reasoning for Elementary Teachers (6th ed.)*	29	5	47	19	100
Miller	*Mathematical Ideas (11th ed.)*	23	19	7	19	68
Musser	*Mathematics for Elementary Teachers: A Contemporary Approach (9th ed.)*	44	16	44	19	123
O'Daffer	*Mathematics for Elementary School Teachers (4th ed.)*	36	5	44	19	104
Parker	*Elementary Geometry for Teachers and Elementary Mathematics for Teachers*	54	24	54	19	151
Rubenstein	*Teaching and Learning Middle Grades Mathematics*	0	16	0	0	16
Sonnabend	*Mathematics for Teachers: An Interactive Approach for Grades K-8 (4th ed.)*	33	7	43	19	102
Sowder	*Reconceptualizing Mathematics*	23	9	30	19	81
Stump	*Algebra for Elementary and Middle School Teachers: An Inquiry Approach*	2	10	0	0	12
Van de Walle	*Elementary and Middle School Mathematics: Teaching Developmentally (8th ed.)*	16	2	5	11	34
Wheeler	*Modern Mathematics for Elementary Educators (12th ed.)*	53	8	20	19	100
Wheeler	*Modern Mathematics for Elementary Educators (13th ed.)*	53	12	20	19	104

Elementary mathematics content textbook reviewers

Information follows on mathematicians (all members of the NCTQ Mathematics Advisory Group) who evaluate elementary content textbooks in conjunction with our evaluations of programs under **Standard 5: Common Core Elementary Mathematics:**

Dr. Richard Askey

Dr. Askey is an emeritus professor at the University of Wisconsin, where he has taught since 1963. He is a Fellow of the American Academy of Arts and Sciences and an Honorary Fellow of the Indiana Academy of Sciences. He was elected to the National Academy of Sciences in 1999.

Professor Askey's research has primarily been in special functions, which are extensions of the functions studied in high school. In addition to many research papers, he coauthored what is now one of the standard books on special functions. More recently he has become involved in issues regarding mathematics education and was on a plenary panel at the 10th International Congress on Mathematics Education. He has reviewed many mathematics education reports both nationally and for various states. He was an Edyth May Sliffe Award winner for his work with high school students.

Dr. Askey received his undergraduate degree from Washington University, his master's degree from Harvard University, and his PhD from Princeton University.

Dr. Andrew Chen

Dr. Chen is the President of EduTron Corporation. Before founding EduTron he was a professor and a principal research scientist at the Massachusetts Institute of Technology. He continues to teach and conduct research in physics. He frequently consults with education research institutions, including the Institute for Education Science at the U.S. Dept. of Education, and Achieve, Inc. Dr. Chen is on the Common Core State Standards Development Team in Mathematics. Locally he is on the Mathematics and Science Advisory Council for the Massachusetts Board of Education.

Dr. Chen provides high quality professional development in mathematics and science to teachers at all levels through Intensive Immersion Institutes. He works with school districts and school administrators to increase their capacity to support excellent mathematics and science instruction. Dr. Chen also works with higher education institutions to develop rigorous and effective preservice and in-service offerings in mathematics and science. He was an Adviser for the Massachusetts 2008 Guidelines for the Mathematical Preparation of Elementary Teachers.

Dr. Chen received a PhD in physics from Columbia University.

Dr. Mikhail Goldenberg

Dr. Goldenberg was a middle school and high school mathematics teacher for three years in Ukraine. Between 1964-1997, he was a professor of mathematics in South Ural State University in Chelyabinsk, Russia. He has worked with advanced high school students in Chelyabinsk Litseum and mathematics teachers at the Institute for Teachers Advance.

Dr. Goldenberg came to the United States in 1997 and became a mathematics teacher for the Ingenuity Project sponsored by the Abell Foundation. He is now the mathematics department head and teaches all the high school mathematics courses. He has led the Ingenuity Math Club for 10 years and is a part-time lecturer at Morgan State University.

Dr. Goldenberg graduated from Odessa State University in 1961 with a master's degree in mathematics and mathematics education. He then received his doctorate degree in Mathematics in 1970 from Ural State University.

Dr. Roger Howe

Dr. Howe has been teaching and conducting research in the Mathematics Department at Yale University for over 35 years. He is currently the William Kenan Jr. Professor of Mathematics. His mathematical research concerns symmetry and its applications. He has held visiting positions at many universities and research institutes in the U.S., Europe, and Asia. He is a member of the American Academy of Arts and Sciences and the National Academy of Sciences.

Dr. Howe devotes substantial attention to issues of mathematics education. He has served on a multitude of committees, including those for several of the major reports on mathematics education of the past decade. He has reviewed mathematics texts and other instructional materials at all levels, from first grade through college. He has served as a member and as chair of the Committee on Education of the American Mathematical Society. He served on the Steering Committee of the Institute of Advanced Study Park City Mathematics Institute, and has helped to organize a series of meetings at Park City devoted to increasing the contribution of mathematicians in mathematics education, especially refining understanding of the mathematical issues in K-12 mathematics curricula. Dr. Howe is currently a member of the U.S. National Committee on Mathematics Instruction and the Executive Commission on Mathematics Education. In 2006, he received the Award for Distinguished Public Service from the American Mathematical Society. He was also a member of the Mathematics Work Team for the Common Core State Standards in Mathematics.

Dr. Howe received his BS in mathematics from Harvard University in 1964, winning the William Lowell Putnam Mathematical Competition. He obtained his PhD from the University of California, Berkeley in 1969.

Dr. R. James Milgram

Dr. James Milgram is an emeritus professor of mathematics at Stanford University where he has taught since 1970. Among other honors, Dr. Milgram has held the Gauss Professorship at the University of Goettingen and the Regents Professorship at the University of New Mexico. He has published over 100 research papers and four books, as well as serving as an editor of many others. His main area of research is algebraic and geometric topology, and he currently works on questions in robotics and protein folding.

Dr. Milgram was a member of the National Board of Education Sciences – the presidential board that oversees the Institute for Education Research at the U.S. Department of Education. He was

also a member of the NASA Advisory Council, the Achieve Mathematics Advisory Panel and a number of other advisory boards. He was one of the members of the Common Ground Project that included Deborah Loewenberg Ball, Joan Ferrini-Mundy, Jeremy Kilpatrick, Richard Schaar, and Wilfried Schmid. From 2002 to 2005, Dr. Milgram headed a project funded by the U.S. Department of Education that identified and described the key mathematics that K-8 teachers need to know. He also helped to direct a project that evaluated state mathematics assessments. He is one of the four main authors of the California mathematics standards, as well as one of the two main authors of the California Mathematics Framework. He is one of the main authors of the Michigan and Georgia K-8 mathematics standards.

Dr. Milgram received his undergraduate and master's degrees in mathematics from the University of Chicago, and his PhD in mathematics from the University of Minnesota.

Dr. Yoram Sagher

Dr. Sagher is professor of mathematics at Florida Atlantic University and emeritus professor of mathematics at the University of Illinois, Chicago. He has written more than 55 research papers in Harmonic Analysis, Real Analysis, and Interpolation Theory and three research papers in mathematics education. Dr. Sagher directed ten doctoral dissertations in mathematics and one in mathematics education. He directed the doctoral dissertation of M.V. Siadat: "Building Study and Work Skills in a College Mathematics Classroom." For his work implementing the methods developed in that paper, Dr. Siadat was named "Illinois Professor of the Year" in 2005 by the Carnegie Foundation.

Dr. Sagher taught numerous continuing education courses for in-service elementary school and high school teachers in Chicago. He also created the course "Methods of Teaching High School Mathematics" at the University of Illinois, Chicago. The course serves as the capstone course for students preparing to become high school mathematics teachers. Dr. Sagher developed highly effective teaching methods that, in combination with the Singapore mathematics textbooks, have produced outstanding results in elementary and middle schools from Boston to Los Angeles, including The Ingenuity Project in Baltimore and Ramona Elementary in Los Angeles. He co-organized two international conferences in mathematics education: Numeracy and Beyond I, Pacific Institute for the Mathematical Sciences at the University of British Columbia, Vancouver, Canada, July 2003, and a follow-up conference, Numeracy and Beyond II, Banff, Canada, December 2004. He provided a week-long intensive workshop to teachers in Trinidad in July 2010. In 2012 he was hired by the World Bank to counsel the Secretary of Education of Rio de Janeiro.

Dr. Sagher received his BS degree from Technion, the Israel Institute of Technology, and his PhD from the University of Chicago.

Non-elementary mathematics content textbooks (not evaluated)

Primary author	Textbook
Aichele	*Geometric Structures: An Inquiry-Based Approach for Prospective Elementary and Middle School Teachers*
Alexander	*Elementary Geometry for College Students*
Angel	*A Survey of Mathematics with Applications*
Ashby	*Introductory and Intermediate Algebra*
Ashlock	*Error Patterns in Computation: Using Error Patterns to Improve Instruction*
Aufmann	*Algebra for College Students (2nd ed.)*
Aufmann	*Mathematical Excursions (3rd ed.)*
Bello	*Topics in Contemporary Mathematics*
Bennett	*Mathematics for Elementary Teachers: An Activity Approach (9th ed.)*
Bennett	*Statistical Reasoning for Everyday Life*
Blitzer	*Thinking Mathematically*
Bluman	*Elementary Statistics: A Step by Step Approach*
Bluman	*Mathematics in Our World*
Boaler	*What's Math Got to Do With It?: How Parents and Teacher Can Help Students Learn to Love Their Least Favorite Subject*
Brase	*Understanding Basic Statistics (6th ed.)*
Browning	*Probability and Statistics for Elementary and Middle School Teachers*
Burger	*The Heart of Mathematics: An Invitation to Effective Thinking*
Burgis	*Investigating College Algebra with Technology*
Burns	*A Collection of Math Lessons, Grades 6-8*
Burns	*Teaching About Mathematics: A K - 8 Resource*
Burris	*Understanding Basic Statistics (6th ed.)*
Burton	*Visual Algebra for College Students*
Carpenter	*Children's Mathematics: Cognitively Guided Instruction*
Carpenter	*Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School*
Cathcart	*Learning Mathematics in Elementary and Middle School: A Learner - centered Approach*
Chapin	*Math Matters: Understanding the Math you Teacher Grades K-8*
Charlesworth	*Math & Science for Young Children*
Coyne	*Effective Teaching Strategies that Accommodate Diverse Learners*
Davis	*Exploring Science and Mathematics in a Child's World*
Dolan	*Mathematics Activities for Elementary School Teachers*
Doxiadius	*Logicomix: An Epic Search for Truth*
Dulyea	*Mathematics Resource Guide*
Eichinger	*Activities for Integrating Science and Mathematics*
Empson	*Extending Children's Mathematics: Fractions and Decimals: Innovations in Cognitively Guided Instruction*
Ensenberger	*The Number Devil: A Mathematical Adventure*
Feikes	*Connecting Mathematics for Elementary Teachers: How Children Learn Mathematics*
Fosnot	*Young Mathematicians at Work: Constructing Number Sense, Addition and Subtraction*
Freitag	*Mathematics for Elementary Teachers (In progress)*

Gies	*Geometry and Problem-Solving*
Ginsberg	*Children's Arithmetic: How They Learn it and How You Teach it*
Gutstein	*Rethinkig Mathematics*
Hardy	*A Mathematician's Apology*
Hatfield	*Mathematics Methods for Elementary and Middle School Teachers (6th ed.)*
Hawkes	*Discovering Statistics (2nd ed.)*
Heddens	*Today's Mathematics: Concepts, Classroom Methods, and Instructional Activities (12th ed.)*
Hiebert	*Making Sense: Teaching and Learning Mathematics with Understanding*
Hudson	*Designing and Implementing Mathematics Instruction for Students with Diverse Learning Needs*
Hutchinson	*Basic Mathematical Skills with Geometry (6th ed.)*
Jacobs	*Mathematics, A Human Endeavor: A book for Those Who Think They Don't Like the Subject*
Johnson	*Crossing the River with Dogs: Problem Solving for College Students*
Johnson	*Elementary Statistics (10th ed.)*
Jones	*Visualizing Elementary and Middle School Mathematics Methods*
Kaplan	*Math on Call*
Kennedy	*Guiding Children's Learning of Mathematics*
Kinsey	*Symmetry, Shape and Space: An Introduction to Math through Geometry*
Kutz	*Foundations of Mathematics I*
Kutz	*Foundations of Mathematics II*
Lamon	*Teaching Fractions and Ratios for Understanding: Essential Content Knowledge and Instructional Strategies for Teachers*
Lappan	*Filling and Wrapping: Three Dimensional Wrapping*
Lemlech	*Curriculum and Instructional Methods for the Elementary and Middle School*
Lial	*Finite Math and Calculus with Applications*
Lial	*Introductory and Intermediate Algebra*
Lial	*Mathematics with Applications*
Martinez	*Teaching Mathematics in Elementary and Middle School: Developing Mathematical Thinking*
Masingila	*Mathematics for Elementary Teachers via Problem Solving: Student Act Manual*
Mason	*Developing Thinking in Algebra*
Math Solutions	*About Teaching: Mathematics: A K-8 Resource*
Mauch	*A Hands On Approach to Geometry and Statistics*
McCarthy	*About Teaching 4MAT in the Classroom*
Mercer	*Teaching Students with Learning Problems*
Moore	*Statistics: Concepts and Controversies (7th ed.)*
Morris	*The Young Child and Mathematics (2nd ed.)*
Murdock	*Discovering Algebra: An Investigative Approach*
Musser	*College Geometry: A Problem Solving Approach with Applications*
NCTM	*Principles and Standards for School Mathematics*
Perkowski	*Data Analysis and Probability Connections: Mathematics for Middle School Teachers*
Pirnot	*Mathematics All Around*
Rachlin	*Algebra I, A Process Approach*
Reimer	*Mathematicians Are People, Too: Stories from the Lives of Great Mathematicians*

Reys	*Helping Children Learn Mathematics (10th ed.)*
Rossman	*Workshop Statistics: Discovery with Data and the Graphing Calculator*
Sarhangi	*Elements of Geometry for Teachers*
Serra	*Discovering Geometry: An Investigative Approach*
Sheffield	*Teaching and Learning Mathematics: Perkindergarten Through Middle School*
Smith	*Improving Instruction in Rational Numbers and Proportionality: Using Cases to Transform Mathemtaics Teaching and Learning*
Smith	*The Nature of Mathematics*
Sousa	*How the Brain Learns Mathematics*
Stein	*Designing Effective Mathematics Instruction: A Direct Instruction Approach*
Tahan	*The Man Who Counted: A Collection of Mathematical Adventures*
Tannenbaum	*Excursions in Modern Mathematics*
Troutman	*Mathematics: A Good Beginning*
Tucker	*Teaching Mathematics to All Children: Designing and Adapting Instructionto Meet the Needs of Diverse Learners (2nd ed.)*
Tussy	*Basic Geometry for College Students: An Overview of the Fundamental Concepts of Geometry (2nd ed.)*
Van de Walle	*Teaching Student Centered Mathematics: Grades K-3*
Warren	*Beginning Statistics*
Wheeler	*Discrete Mathematics for Teachers*