Understanding Our Elementary Mathematics Standard

The program prepares teacher candidates to teach to increasingly rigorous state standards for elementary math.

WHY THIS STANDARD?

After teaching reading, the most important job for elementary and special education teachers is to establish a strong foundation in mathematics. Not only will students with this foundation have a much better chance of succeeding in school, but we can increase the number of students with the skills necessary to consider all-important STEM (Science, Technology, Engineering and Math) careers.

WHAT IS THE FOCUS OF THE STANDARD?

The standard evaluates the specialized coursework elementary and special education teachers should take in order to gain the deep conceptual understanding of elementary math topics required to teach to increasingly rigorous state standards. Programs meeting this standard in full not only require strong math content courses but also a math methods course in how to teach math. A program earning a “strong design” designation does a particularly good job coordinating math content and methods coursework.

Standard applies to: Elementary and Special Education programs.

Standard and Indicators

Standard 5: Elementary Mathematics

The program prepares teacher candidates to teach to increasingly rigorous state student learning standards for elementary math.

Standard applies to: Elementary and Special Education programs.

Indicators that the program meets the standard:

5.1 As a condition of admission, the program requires that elementary teacher candidates show proficiency in elementary math content topics by passing a single rigorous elementary math content exam or by passing the math section of a rigorous elementary exam with separate cut-scores provided for all subjects (such as the Praxis II Elementary Education: Multiple Subjects test).

OR

5.2 Programs require candidates to take a course sequence that thoroughly covers essential elementary mathematics topics in numbers and operations, algebra, geometry, and data analysis.

AND

5.3 These courses use appropriate textbooks and support instruction on essential topics of elementary mathematics.

AND

5.4 The program requires an adequate elementary mathematics methods course.

OR

5.5 The program satisfies indicators 5.2-5.4 in large part and also produces graduates whose students' performance in mathematics is sufficiently superior to the performance of their peers to generate statistically significant positive results in the two most recent program-specific state reports.

Indicators that the program has strong design:

5.5 A program will earn a "strong design" designation if adequate elementary mathematics content is combined with elementary mathematics methods instruction in a coordinated set or sequence of courses that satisfies indicators 5.2-5.4.

Rationale

Standard 5: Elementary Mathematics

The program prepares teacher candidates to teach to increasingly rigorous state standards for elementary math.

Standard applies to: Elementary and Special Education programs.

WHY THIS STANDARD?

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RATIONALE

Research base for this standard

"Strong research"[1] shows that, in general, students achieve more in math when taught by teachers with greater mathematics content knowledge.[2] A study of teacher preparation programs (both traditional and alternative) in New York City found that math courses correlated with increased student achievement in math during the second year of teaching.[3] Another study found no correlation between teachers' math education credits and student achievement in math.[4]

1 NCTQ has created "research inventories" that describe research conducted within the last decade or so that has general relevance to aspects of teacher preparation also addressed by one or more of its standards (with the exceptions of the Outcomes, Evidence of Effectiveness, and Rigor standards). These inventories categorize research along two dimensions: design methodology and use of student performance data. Research that satisfies our standards on both is designated as "strong research" and will be identified as such. That research is cited here if it is directly relevant to the standard; strong research is distinguished from other research that is not included in the inventory or is not designated as "strong" in the inventory. Refer to the introduction to the research inventories for more discussion of our approach to categorizing research. If a research inventory has been developed to describe research that generally relates to the same aspect of teacher prep as addressed by a standard, the inventory can be found in the back of this standard book.

2 Campbell, P. F., Nishio, M., Smith, T. M., Clark, L. M., Conant, D. L., Rust, A. H., DePiper, J. N., Frank, T. J., Griffin, M. J., & Choi, Y. (2014). The relationship between teachers' mathematical content and pedagogical knowledge, teachers' perceptions, and student achievement. *Journal for Research in Mathematics Education, 45*(4), 419-459; Kukla-Acevedo, S. (2009). Do teacher characteristics matter? New results on the effects of teacher preparation on student achievement. *Economics of Education Review, 28*, 49-57; Hill, H., Rowan, B., & Ball, D. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371-406. Blazar (2015) found a relationship between teachers' mathematical content knowledge and two instructional characteristics: ambitiousness of instruction and frequency of errors and imprecision. While these characteristics also related to student achievement, the study did examine the direct relationship between teachers' math content knowledge and student achievement. Blazar, D. (2015). Effective teaching in elementary mathematics: Identifying classroom practices that support student achievement. *Economics of Education Review, 48*, 16-29.

3 Boyd, D. J., Grossman, P. L., Lankford, H., Loeb, S., & Wyckoff, J. (2009). Teacher preparation and student achievement. *Educational Evaluation and Policy Analysis, 31*(4), 416-440. This study notes that its findings may differ from those in Harris & Sass (2011) because the Boyd study looked at "data on the characteristics of programs, courses, and field experiences," while the Harris study used course credit hours and hours of in-service training.

4 Harris, D. N., & Sass, T. R. (2011). Teacher training, teacher quality and student achievement. *Journal of Public Economics, 95*, 798-812. Note: This study relates to several NCTQ standards. Although it meets the criteria for strong research, the study's findings run contrary to the conclusions of most strong research in the field.

Content coursework in elementary mathematics provides a foundation for essential methods coursework, as math content knowledge is a prerequisite for the even more difficult tasks requiring pedagogical content knowledge, such as predicting and identifying student errors.[5]

Taken as a whole, the preponderance of additional available research studies[6] indicates that the mathematics content coursework needed by elementary teachers is neither pure mathematics nor pure methods but a combination of both. This mix of coursework imparts the foundational knowledge of elementary mathematics topics and is a bridge to instruction in the elementary classroom.[7]

Other studies have found that students performed better when their teachers had taken a mathematics course designed for teacher candidates, rather than a pure mathematics course,[8] and that higher-level mathematics courses (past calculus) had detrimental effects on student learning nearly as often as they had positive effects.[9]

Research on mathematics methods, although limited, also indicates the value of mathematics methods courses.[10] Research generally supports the importance of teachers' knowledge of fundamental math concepts as well as their ability to apply mathematics content in teaching (learned in mathematics methods courses), rather than their just knowing the mathematics content.[11]

Other support for this standard

Detailed surveys of elementary teachers in 60 school districts in Michigan and Ohio[12] indicate that elementary teachers do not feel well prepared to teach the specific mathematics topics at the elementary level or slightly beyond:

- In at least three-quarters of the districts, less than half of first through third-grade teachers considered themselves "very well-prepared" to teach more than 60 percent of the topics. A little more than half of teachers in all districts felt "very well-prepared" to teach just three topics.
- In at least three-quarters of the districts, less than half of fourth and fifth-grade teachers considered themselves "very well-prepared" to teach more than 50 percent of the topics. At least 55 percent of teachers in all districts only felt "very well-prepared" to teach just three topics.

There is increasing consensus among mathematics education organizations that prospective elementary teachers would be more prepared to teach elementary math if they took college mathematics courses that were designed specifically

5 Harris, D. N., & Sass, T. R. (2011). Teacher training, teacher quality and student achievement. *Journal of Public Economics, 95*, 798-812. Note: This study relates to several NCTQ standards. Although it meets the criteria for strong research, the study's findings run contrary to the conclusions of most strong research in the field.

6 "Additional research" is research that is not designated as "strong" because it is not as recent and/or does not meet the highest standards for design methodology and/or use of student performance data.

7 Greenberg, J., & Walsh, K. (2008). No Common Denominator. Washington, DC: National Council on Teacher Quality.

8 Monk, D. (1994). Subject area preparation of secondary mathematics and science teachers and student achievement. *Economics of Education Review, 13*(2), 125-145.

9 Begle, E. G. (1970). Critical variables in mathematics education: Findings from a survey of empirical literature. Washington, DC: Mathematical Association of America and National Council of Teachers of Mathematics.

10 Promoting Rigorous Outcomes in Mathematics and Science Education (2006, December). Knowing mathematics: What we can learn from teachers (Research Report, Vol. 2). East Lansing, MI: Michigan State University.

11 Ball, D., Lubienski, S., & Mewborn, D. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), Handbook on research on teaching (4th ed.). Washington, DC: American Educational Research Association; Guyton, E., & Farokhi, E. (1987). Relationships among academic performance, basic skills, subject matter knowledge, and teaching skills of teacher education graduates. *Journal of Teacher Education, 38*, N5.

12 Schmidt, W. H., & McKnight, C. (2002). Inequality for all: The challenge of unequal opportunity in American schools. New York: Teachers College Press, Columbia University.

for teachers and that imparted a deep understanding of elementary and middle school mathematics concepts.[13] These recommendations follow the prevailing research on teacher preparation in mathematics, as described in the research base for this standard. A calculus or statistics course is fine as an elective, but the National Council of Teachers of Mathematics (NCTM)[14] and the Conference Board of the Mathematical Sciences (CBMS)[15] recommend that aspiring elementary teachers also take 12 semester-credit hours in "elementary mathematics content." This content should cover four subject areas: numbers and operations, algebra, measurement and data, and geometry. These recommendations, as well as those of mathematicians who advised NCTQ in our national study of the mathematics preparation of elementary teachers, form the rationale for this standard with regard to content preparation.

As in early reading, the majority of state licensing tests for elementary candidates are insufficiently rigorous to determine if candidates have sufficiently mastered math to be effective elementary teachers, making an evaluation of coursework necessary. However, states have made progress in the last few years. Now, 28 states have rigorous tests that either test mathematics as a standalone subject or provide a specific mathematics subscore. Other states rely on subject-matter tests that include some items (or even a whole section) on mathematics instruction. However, since subject-specific passing scores are not required, one could answer every mathematics question incorrectly and still pass. The content of these tests presents another issue: These tests should assess whether candidates have a sophisticated understanding of the content they will teach in elementary and middle grades, not whether they simply meet the state's expectations for *student* knowledge of math. Unfortunately, this level of difficulty is not present in the tests most states currently use.

Teacher preparation programs in high-achieving nations frequently ensure that teachers not only know the content but also can communicate it. Mathematics-specific pedagogy is part of the preparation of mathematics teachers around the world, including in countries such as Singapore, Korea, and Taiwan, whose students outperform our own.[16] An analysis of courses taken by elementary teachers across 17 countries found five commonly required mathematics courses and another seven common mathematics electives addressing both content and pedagogy.[17]

The methodology includes assessing the content and quality of mathematics textbooks as part of the rating process for this standard. This application of textbook ratings is supported by a recent study by the National Center for Education Statistics, which evaluated the rigor of mathematics textbooks as a proxy for evaluating the rigor of individual mathematics courses.[18] This study proffers that textbooks can indicate the intended course curriculum, the skills and topics taught in a course, and the level of rigor of that course. NCTQ relies on this same understanding of the role of textbooks in its methodology for this standard.

This standard also draws support from school district superintendents.

13 In this vein, a University of Virginia professor of psychology recently argued that elementary teachers need to be trained to understand and teach the "conceptual side of math," or else they cannot build a strong math foundation for their young students. Willingham, D. (2013). What the NY Times doesn't know about math instruction. Retrieved March 12, 2014 from www.DanielWillingham.com

14 National Council of Teachers of Mathematics. (2005, July). Highly qualified teachers: A position of the National Council of Teachers of Mathematics (Position Paper). Retrieved January, 2011, from http://www.nctm.org/about/content.aspx?id=6364.

15 American Mathematical Society in Cooperation with the Mathematical Association of America. (2012). *The Mathematical Education of Teachers II, 17*. Retrieved February, 2013, from http://cbmsweb.org/MET2/index.htm

16 Jensen, B., Roberts-Hull, K., Magee, J., & Ginnivan, L. (2016). *Not so elementary: Primary school teacher quality in high-performing systems*. Washington, DC: National Center on Education and the Economy. Communications with Mdm. Low Khah Gek, Deputy Director, Sciences, Curriculum Planning and Development Division, Ministry of Education, Singapore, March 2008.

17 Schmidt, W., Burroughs, N., Cogan, L. (2013). World class standards for preparing teachers of mathematics (Working Paper). East Lansing, MI: Michigan State University Center for the Study of Curriculum and The Education Policy Center.

18 Brown, J., Schiller, K., Roey, S., Perkins, R., Schmidt, W., & Houang, R. (2013). Algebra I and Geometry Curricula (NCES 2013-451). U.S. Department of Education, National Center for Education Statistics. Washington, DC: U.S. Government Printing Office. Retrieved March 18, 2013, from http://nces.ed.gov/nationsreportcard/pdf/studies/2013451.pdf

Methodology

How NCTQ scores the Elementary Mathematics Standard

Standards and Indicators

DATA USED TO SCORE THIS STANDARD

Evaluation of elementary and special education programs on Standard 5: Elementary Mathematics uses the following sources of data:

- Course descriptions of elementary mathematics content and methods courses in institution of higher education (IHE) catalogs
- Elementary math content and methods course credit information in IHE catalogs
- Syllabi of required elementary math content courses
- Required primary textbooks in required elementary math content courses
- Integrated Postsecondary Education Data System (IPEDS) data on mean university SAT/ACT scores and mean of SAT/ACT scores self-reported to the College Board[19] (undergraduate), or requirement of the Graduate Records Examination (GRE) (graduate)
- Pre-admission tests that require a separate cut-score in elementary math
- Value-added data on teachers who graduated from the program

WHO ANALYZES THE DATA

One subject specialist evaluates each program using a detailed scoring protocol from which this scoring methodology is abstracted. Twenty percent of programs are randomly selected for a second evaluation to assess scoring variances. For information on the process by which scoring discrepancies are resolved, see the "scoring processes" section of the General Methodology.

SCOPE OF ANALYSIS

Scores of **undergraduate** and **graduate** teacher preparation programs on elementary math preparation are based on examination of course descriptions, syllabi, and required primary textbooks in coursework designed for teacher audiences. (A discussion of the use of syllabi and textbooks for analysis of course content is provided here.) For graduate programs, transcript review processes are evaluated to ascertain whether they are sufficiently prescriptive about the need for elementary math content coursework or simply ensure that applicants have completed some amount of college-level math coursework.

Required textbooks are evaluated in each of four "critical subject areas" (numbers and operations, algebra, geometry, and data analysis) by mathematicians according to a specific protocol.[20] Ratings of reviewed elementary math textbooks are provided here.

19 Used if more than 50 percent of the student body report such scores and no other source of SAT/ACT data is available.

20 The evaluators assess the topics in each critical area on the basis of coverage, connection, integrity, the sufficiency and significance of examples, and whether the text addresses methods of teaching.

Analysts score syllabi based on coverage in lectures, assignments, and/or assessments of 12 "essential topics" subsumed within those four critical subject areas.[21] For credit to be awarded, mention of a topic or subtopic is needed only in a course topic outline, a listing of lectures, or a listing of assignments; no textbook coverage of a topic is necessary. However, because of the many subtopics that the evaluation looks for in lectures and assignment — more than would be found even in the most detailed syllabus — the analysis is enhanced to encompass the full breadth of classroom instruction by consulting the sections of the textbook to which the syllabus explicitly connects that instruction.[22]

In accordance with Indicators 5.1 and 5.2, classroom instruction scores for each of the essential topics and textbook scores for each course are used to develop a composite syllabus/textbook score that is then averaged to produce a "cumulative score" for the program as a whole. Topics addressed by the program that are not included in those identified as essential are classified as "other"; one-half of the proportion of such coverage (up to 10 percent) is added to the "cumulative score" for a final "instructional score." The instructional score must be 75 percent or higher for a program to at least partly meet the standard.

Common misconceptions about how analysts evaluate the Elementary Mathematics Standard:

- *Any math content course required of teacher candidates is relevant for evaluation for this standard.* It may be advisable for teacher candidates to take a variety of math courses additional to those on elementary math — as it would be for any undergraduate or graduate student. However, this standard evaluates only elementary math content coursework because of the unique value this coursework provides for professional preparation.

- *Elementary math methods coursework that addresses content is interchangeable with elementary content coursework.* Elementary math methods course should be grounded in an understanding of the relevant content, and references to that content are essential to full development of pedagogy. However, even the most content-infused math methods course does not substitute for elementary math content coursework.

- *Math content coursework designed for both elementary and middle school teacher candidates provides adequate instruction for each type of candidate.* While both elementary and middle school teacher candidates need to have some grasp of the content relevant to the full K-8 grade span, they will not develop the conceptual understanding necessary for adequate instruction without the benefit of coursework focusing on the grades they will teach.

Overall program scores factor in the instructional score of the courses and the number of required course credits in elementary math[23] (as determined by evaluation of course descriptions and course titles in the context provided by state certification grade spans[24]). As precise an estimate as possible of the semester credit hours (SCHs) devoted to elementary

21 Whole numbers and place value; fractions and integers; decimals (including ratio, proportion, percent); estimation; constants, variables, expressions; equations; graphs and functions; measurement; basic concepts in plane and solid geometry; polygons and circles; perimeter, area, surface area, volume; probability and data display and analysis.

22 Samples of course syllabi and their evaluation are contained in Appendix E of NCTQ's national mathematics report which can be accessed at http://www.nctq.org/p/publications/docs/nctq_ttmath_fullreport_20090603062928.pdf

23 We note that non-elementary math coursework (such as a college algebra course) is not relevant for this standard.

24 For example, unless otherwise indicated in the course description, an "elementary" math content course is presumed to provide instruction designed for both elementary and middle school teacher candidates in a state in which elementary teacher certification spans grades K–8. A course in the same state whose description contains a significant number of references to pedagogical and content concerns would be presumed to be designed for both elementary and middle school teacher candidates to address both elementary methods and content and middle school methods and content.

We note that a course designed for elementary teacher candidates should address both elementary and middle school math content but should do so in a manner appropriate for elementary teacher candidates, not for elementary and middle school teacher candidates. Elementary certification grade spans in each state are found in Teacher Licensing Structure Infographics.

math content relevant to the K–5 teacher[25] is used. Elementary math methods coursework is relevant only to determine if an elementary program[26] fully satisfies the requirement for adequate elementary content instruction.[27]

Programs with instructional scores below 60 percent or requiring three or fewer SCHs of elementary math coursework do not meet the standard. For Indicator 5.3, elementary programs meeting the standard with regard to instructional scores and required coursework credits are evaluated for adequacy in math methods coursework[28] to determine if they meet the standard.

A program can also meet the elementary standard through two means in addition to evaluating its elementary mathematics coursework and textbooks: First, a program can meet this standard if it requires for admission a test providing a separate cut score for elementary mathematics content and the cut score is set at a level recommended by the test developer or its state.

Second, some states now produce value-added data that links teachers' performance back to their preparation programs. If aggregated value-added data for a program's graduates show that they earn above-average value added results in two consecutive state reports, that information will be considered in conjunction with the nature of elementary math content coursework to determine that the program either nearly meets or meets the standard.

If coverage of essential topics cannot be determined in the review of syllabi, analysts attempt to evaluate the program using the alternate scoring process outlined below.

How a program earns a "strong design" rating

Evaluation of programs for strong design entails a review of how programs that fully satisfy the standard for strong elementary math preparation coordinate or integrate elementary math content and methods coursework.

An alternate scoring process if data are not provided

Because elementary preparation is critical to ensuring that elementary and special education teacher candidates are competent to enter the classroom, NCTQ could not allow the lack of cooperation on the part of IHEs to place them out of the reach of evaluations on this standard. To that end, a means of evaluating elementary and special education programs on this standard using imputation was devised after extensive field work.[29]

This imputation process relies on the following sources of data:

- Course descriptions of elementary math content and methods courses in institution of higher education (IHE) catalogs
- Elementary math content and methods course credit information in IHE catalogs
- Listings in IHE bookstores of required primary textbooks in required elementary math content courses
- Integrated Postsecondary Education Data System (IPEDS) data on mean university SAT/ACT scores and mean of SAT/ACT scores self-reported to the College Board[30] (undergraduate) or requirement of the Graduate Records Examination (GRE; graduate)

25 This includes content for grades K–5 and also some coverage of middle school math topics.

26 Special education programs can fully satisfy the standard based on consideration of only math content requirements, with no consideration of math methods coursework requirements.

27 "Adequacy" is defined differently based on the selectivity of the IHE in which the program is housed: At least six SCHs for undergraduate programs in IHEs whose student body's mean math and verbal SAT/composite ACT scores are at or above 1120/24 respectively or for graduate programs that require the GRE for admission; at least eight SCHs for programs in all other IHEs.

28 At least three SCHs.

29 We estimate that in 80 percent of programs, imputation produces the same program scores as evaluation with complete data.

30 Used if more than 50 percent of the student body report such scores and no other source of SAT/ACT data is available.

The fundamental difference between the two scoring approaches is that the instructional score produced by imputation uses course descriptions and textbook evaluations rather than course descriptions, textbook evaluations, and syllabi.

Scores produced by imputation are reported as Pass (3.5 on a 0-4 scale) or Fail (1 on a 0-4 scale).

Any program that could not be evaluated by either the standard scoring process or the alternate scoring process was removed from the sample.

Examples of what satisfies or does not satisfy the standard's indicators

Adequate course sequence supported by textbooks and including a methods course (Indicators 5.1–5.4)

✓ fully satisfies the indicators	✗ does not satisfy the indicators
In an undergraduate elementary program housed in a sufficiently selective IHE… Coursework designed for grades K–5 teacher candidates satisfies Indicators 5.1 – 5.4: ■ It earns an instructional **score of 86 percent**; ■ Nine SCHs of combined math content and methods coursework — **six SCHs of elementary math content and three SCHs of math methods** — are required. *Note: Eight SCHs of elementary math content coursework are required to satisfy Indicator 5.1 unless, as is the case in this program, the IHE student body's mean combined math and verbal SAT is at or above 1120 (or its composite ACT is at or above 24). In such cases, only six SCHs are required. Indicator 5.3 is satisfied by the three SCHs of methods instruction in the combined math content and methods coursework.*	In a graduate program that is not sufficiently selective… Coursework designed for grades K–5 teacher candidates satisfies only Indicator 5.4: ■ No elementary math content coursework is required of teacher candidates; ■ Three SCHs of elementary math methods coursework are required. *Note: Because it does not signify that it is selective by requiring the GRE for admission, this program would have to require at least eight SCHs of elementary math content designed for grades K–5 teacher candidates to have the potential to satisfy the standard either partly or fully. While valuable, the math methods course does not satisfy the standard.*
In a graduate elementary program, the transcript review during the admission process specifies that applicants must have completed an elementary math course. For example: ■ CI 5822: Teaching Mathematics in Elementary School – must be taken as part of the undergraduate curriculum or as part of the first summer session.	In a graduate elementary program: ■ The transcript review during the admission process makes no reference to math; ■ The transcript review during the admission process asks candidates to submit evidence of college-level work in mathematics, such as courses in algebra, analytic geometry, and calculus.

Research Inventory

Researching Teacher Preparation: Studies investigating the preparation of teacher candidates for elementary mathematics instruction

These studies address issues most relevant to Standard 5: Elementary Mathematics

Total number of studies	Studies with stronger design		Studies with weaker design	
	Measures student outcomes	**Does not measure student outcomes**	**Measures student outcomes**	**Does not measure student outcomes**
44	**8**	**12**	**0**	**24**
	Citations: 4-6, 11, 12, 16, 20, 21	Citations: 1-3, 9, 19, 26, 32, 33, 35, 36, 39, 41		Citations: 7, 8, 10, 13–15, 17, 18, 22–25, 27-31, 34, 37, 38, 40, 42-44

Note: Boyd et al. (2009) is cross-listed with RI 6: Elementary Content and RI 14: Student Teaching; Kukla-Acevedo (2009) is cross-listed with RI 1: Selection Criteria; and Livy & Vale (2011) is cross-listed with RI 9: Content for Special Education.

Citations for articles categorized in the table are listed below.

Databases: Education Research Complete and Education Resource Information Center (peer-reviewed listings of reports on research including United States populations, with the exception of citation 24 which is not peer-reviewed but is included because of the research experience of the author).

Publication dates: Jan 2000 – August 2016

See Research Inventories: Rationale and Methods for more information on the development of this inventory of research.

1. Aslan-Tutak, F., & Adams, T. L. (2015). A study of geometry content knowledge of elementary preservice teachers. *International Electronic Journal of Elementary Education, 7*(3), 301-318.
2. Berk, D., & Hiebert, J. (2009). Improving the mathematics preparation of elementary teachers, one lesson at a time. *Teachers and Teaching: Theory and Practice, 15*(3), 337–356.
3. Beswick, K., & Goos, M. (2012). Measuring pre-service primary teachers' knowledge for teaching mathematics. *Mathematics Teacher Education and Development, 14*(2), 70-90.
4. Blazar, D. (2015). Effective teaching in elementary mathematics: Identifying classroom practices that support student achievement. *Economics of Education Review, 48*, 16-29.
5. Boyd, D. J., Grossman, P. L., Lankford, H., Loeb, S., & Wyckoff, J. (2009). Teacher preparation and student achievement. *Educational Evaluation and Policy Analysis, 31*(4), 416–440.
6. Campbell, P. F., Nishio, M., Smith, T. M., Clark, L. M., Conant, D. L., Rust, A. H., DePiper, J. N., Frank, T. J., Griffin, M. J., & Choi, Y. (2014). The relationship between teachers' mathematical content and pedagogical knowledge, teachers' perceptions, and student achievement. *Journal for Research in Mathematics Education, 45*(4), 419-459.

7. Chamberlin, M. T., & Chamberlin, S. A. (2010). Enhancing preservice teacher development: Field experiences with gifted students. *Journal for the Education of the Gifted, 33*(3), 381–416.

8. Cramer, K. (2004). Facilitating teachers' growth in content knowledge. *Yearbook* (National Council of Teachers of Mathematics), *66*, 180–194.

9. Depaepe, F., Torbeyns, J., Vermeersch, N., Janssens, D., Janssen, R., Kelchtermans, G., Verschaffel, L., Van Dooren, W. (2015). Teachers' content and pedagogical content knowledge on rational numbers: A comparison of prospective elementary and lower secondary school leaders. *Teaching and Teacher Education, 47*, 82-92.

10. Ford, P., & Strawhecker, J. (2011). Co-teaching math content and math pedagogy for elementary pre-service teachers: A pilot study. *Issues in the Undergraduate Mathematics Preparation of School Teachers, 2*.

11. Goldhaber, D. (2007). Everyone's doing it, but what does teacher testing tell us about teacher effectiveness? *Journal of Human Resources, 42*(4), 765–794.

12. Harris, D. N., & Sass, T. R. (2011). Teacher training, teacher quality and student achievement. *Journal of Public Economics, 95*, 798–812.

13. Hart, L. C., & Swars, S. L. (2009). The lived experiences of elementary prospective teachers in mathematics content coursework. *Teacher Development, 13*(2), 159–172.

14. Herron, J. (2010). Implementation of technology in an elementary mathematics lesson: The experiences of pre-service teachers at one university. *SRATE Journal, 19*(1), 22–29.

15. Hertzog, H. S., & O'Rode, N. (2011). Improving the quality of elementary mathematics student teaching: Using field support materials to develop reflective practice in student teachers. *Teacher Education Quarterly, 38*(3), 89–111.

16. Hill, H.C., Rowan, B., & Ball, D.L. (2005). Effects of teachers' mathematical knowledge for teaching on students' achievement. *American Educational Research Journal, 42*(2), 371–406.

17. Kajander, A. (2010). Elementary mathematics teacher preparation in an era of reform: The development and assessment of mathematics for teaching. *Canadian Journal of Education, 33*(1), 228–255.

18. Kirtman, L. (2008). Pre-service teachers and mathematics: The impact of service-learning on teacher preparation. *School Science and Mathematics, 108*(3), 94–102.

19. Kleickmann, T., Richter, D., Kunter, M., Elsner, J., Besser, M., Krauss, S., Cheo, M., Baumert, J. (2015). Content knowledge and pedagogical content knowledge in Taiwanese and German mathematics teachers. *Teaching and Teacher Education, 46*, 115-126.

20. Kukla-Acevedo, S. (2009). Do teacher characteristics matter? New results on the effects of teacher preparation on student achievement. *Economics of Education Review, 28*(1), 49–57.

21. Li, X., Ding, M., Capraro, M., & Capraro, R. M. (2008). Sources of differences in children's understandings of mathematical equality: Comparative analysis of teacher guides and student texts in China and the United States. *Cognition and Instruction, 26*(2), 195–217.

22. Livy, S., & Vale, C. (2011). First year pre-service teachers' mathematical content knowledge: Methods of solution for a ratio question. *Mathematics Teacher Education and Development, 13*(2), 22–43.

23. Marbach-Ad, G., & McGinnis, J. (2009). Beginning mathematics teachers' beliefs of subject matter and instructional actions documented over time. *School Science and Mathematics, 109*(6), 338–354.

24. Matthews, M. E., & Seaman, W. I. (2007). The effects of different undergraduate mathematics courses on the content knowledge and attitude towards mathematics of preservice elementary teachers. *Issues in the Undergraduate Mathematics Preparation of School Teachers, 1*.

25. Matthews, M., Rech, J., & Grandgenett, N. (2010). The impact of content courses on pre-service elementary teachers' mathematical content knowledge. *Issues in the Undergraduate Mathematics Preparation of School Teachers, 1*.

26. McCrory, R., Zhang, C., Francis, A., & Young, S. (2009). Factors in the achievement of preservice elementary teachers in mathematics classes. In proceedings of the 31st annual meeting of the North American chapter of the international group for the psychology of mathematics education, 5, 1179–1187.

27. McGinnis, J., Watanabe, T., & McDuffie, A. (2005). University mathematics and science faculty modeling their understanding of reform based instruction in a teacher preparation program: Voices of faculty and teacher candidates. *International Journal of Science and Mathematics Education, 3*(3), 407–428.

28. McLeod, K., & Huinker, D. (2007). University of Wisconsin-Milwaukee mathematics focus courses: Mathematics content for elementary and middle grades teachers. *International Journal of Mathematical Education in Science and Technology, 38*(7), 949–962.

29. Mizell, J. A., & Cates, J. (2004). The impact of additional content courses on teacher candidates' beliefs regarding mathematics content and pedagogy. *Issues in the Undergraduate Mathematics Preparation of School Teachers, 4*, 1–11.

30. Moseley, C., & Utley, J. (2006). The effect of an integrated science and mathematics content-based course on science and mathematics teaching efficacy of preservice elementary teachers. *Journal of Elementary Science Education, 18*(2), 1–12.

31. Norton, S. (2010). How deeply and how well? How ready to teach mathematics after a one-year program? *Mathematics Teacher Education and Development, 12*(1), 65–84.

32. Philipp, R. A., Ambrose, R., Lamb, L. C., Sowder, J. T., Schappelle, B. P., Sowder, L., & ... Chauvot, J. (2007). Effects of early field experiences on the mathematical content knowledge and beliefs of prospective elementary school teachers: An experimental study. *Journal for Research in Mathematics Education, 38*(5), 438–476.

33. Pratt, E. (2007). Teacher work samples. *International Journal of Learning, 13*(11), 99–106.

34. Richardson, G. M., & Liang, L. L. (2008). The use of inquiry in the development of preservice teacher efficacy in mathematics and science. *Journal of Elementary Science Education, 20*(1), 1–16.

35. Schmidt, W. H., Cogan, L., & Houang, R. (2011). The role of opportunity to learn in teacher preparation: An international context. *Journal of Teacher Education, 62*(2), 138–153.

36. Schmidt, W. H., Burroughs, N. A., Cogan, L. S., & Houang, R. T. (2016). The role of subject-matter content in teacher preparation: an international perspective. *Journal of Curriculum Studies*, 1-21.

37. Seaman, C. E., Szydlik, J., Szydlik, S. D., & Beam, J. E. (2005). A comparison of preservice elementary teachers' beliefs about mathematics and teaching mathematics: 1968 and 1998. *School Science and Mathematics, 105*(4), 197–210.

38. Sezer, R. (2008). Integration of critical thinking skills into elementary school teacher education courses in mathematics. *Education, 128*(3), 349–362.

39. Smith, M. E., Swars, S. A., Smith, S. Z., Hart, L. C., & Haardörfer, R. (2012). Effects of an Additional Mathematics Content Course on Elementary Teachers' Mathematical Beliefs and Knowledge for Teaching. *Action in Teacher Education, 34*(4), 336 - 348.

40. Spitzer, S., Phelps, C., Beyers, J., Johnson, D., & Sieminski, E. (2011). Developing prospective elementary teachers' abilities to identify evidence of student mathematical achievement. *Journal of Mathematics Teacher Education, 14*(1), 67–87.

41. Strawhecker, J. (2005). Preparing elementary teachers to teach mathematics: How field experiences impact pedagogical content knowledge. *Issues in the Undergraduate Mathematics Preparation of School Teachers, 4*.

42. Swars, S., Hart, L. C., Smith, S. Z., Smith, M. E., & Tolar, T. (2007). A longitudinal study of elementary pre-service teachers' mathematics beliefs and content knowledge. *School Science and Mathematics, 107*(8), 325–335.

43. Swars, S. L., Smith, S. Z., Smith, M. E., & Hart, L. C. (2009). A longitudinal study of effects of a developmental teacher preparation program on elementary prospective teachers' mathematics beliefs. *Journal of Mathematics Teacher Education, 12*(1), 47–66.

44. Zevenbergen, R. (2004). Study groups as a tool for enhancing preservice students' content knowledge. *Mathematics Teacher Education and Development, 6*, 3–20.

National Council on Teacher Quality

1120 G Street, NW, Suite 800
Washington, D.C. 20005
Tel: 202 393-0020 Fax: 202 393-0095
Web: www.nctq.org