No Common Denominator

The Preparation of Elementary Teachers in **Mathematics** by America's Education Schools

Executive Summary
June 2008

NATIONAL COUNCIL ON TEACHER QUALITY

The full report of *No Common Denominator: The Preparation of Elementary Teachers in Mathematics by America's Education Schools* is available online from www.nctq.org.

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EXECUTIVE SUMMARY

In this second study of education schools,[1] the National Council on Teacher Quality (NCTQ) examines the mathematics preparation of America's elementary teachers.[2] The impetus for this study is the mediocre performance of American students in mathematics compared to their counterparts around the world. Though improving American students' relative performance depends on a variety of factors, a particularly critical consideration must be the foundations laid in elementary school because mathematics relies so heavily on cumulative knowledge. The link from there to the capability of elementary teachers to provide effective instruction in mathematics is immediate. Unfortunately, by a variety of measures, many American elementary teachers are weak in mathematics and are too often described, both by themselves and those who prepare them, as "math phobic."

Absent a conclusive body of research on how best to prepare elementary teacher candidates, we devoted two years of study to develop a set of five standards that would be the mark of a high quality program of teacher training. To ensure that these standards were well-founded and comprehensive, we consulted:

- Our own Mathematics Advisory Group, consisting of mathematicians and distinguished teachers with a long history of involvement in K-12 education.
- The recommendations of professional associations, in particular the National Council on Teachers of Mathematics (NCTM), as well as the best state standards, and other key national studies.
- Numerous mathematicians, mathematics educators, cognitive psychologists, social scientists, and economists.
- Education ministries of other nations with higher performance in mathematics than our own, in particular Singapore, whose students lead the world in mathematics performance.

1 While teacher preparation programs do not always reside in "education schools," we refer to them as education schools because the phrase is commonly understood.

2 In May 2006 we issued *What Education Schools Aren't Teaching about Reading and What Elementary Teachers Aren't Learning* http://www.nctq.org/p/publications/docs/nctq_reading_study_app_20071202065019.pdf

Five Standards for the Mathematics Preparation of Elementary Teachers

STANDARD 1:

Aspiring elementary teachers must begin to acquire a deep conceptual knowledge of the mathematics that they will one day need to teach, moving well beyond mere procedural understanding. Required mathematics coursework should be tailored to the unique needs of the elementary teacher both in design and delivery, focusing on four critical areas:

1. numbers and operations,
2. algebra,
3. geometry and measurement, and — to a lesser degree —
4. data analysis and probability.

STANDARD 2:

Education schools should insist upon higher entry standards for admittance into their programs. As a condition for admission, aspiring elementary teachers should demonstrate that their knowledge of mathematics is at the high school level (geometry and coursework equivalent to second-year algebra). Appropriate tests include standardized achievement tests, college placement tests, and sufficiently rigorous high school exit tests.

STANDARD 3:

As conditions for completing their teacher preparation and earning a license, elementary teacher candidates should demonstrate a deeper understanding of mathematics content than is expected of children. Unfortunately, no current assessment is up to this task.

STANDARD 4:

Elementary content courses should be taught in close coordination with an elementary mathematics methods course that emphasizes numbers and operations. This course should provide numerous opportunities for students to practice-teach before elementary students, with emphasis placed on the delivery of mathematics content.

STANDARD 5:

The job of teaching aspiring elementary teachers mathematics content should be within the purview of mathematics departments. Careful attention must be paid to the selection of instructors with adequate professional qualifications in mathematics who appreciate the tremendous responsibility inherent in training the next generation of teachers and who understand the need to connect the mathematics topics to elementary classroom instruction.

This study evaluates the elementary education programs at a sample of 77 education schools located in every state except Alaska. The schools did not volunteer to participate in this study, but were notified early on that they had been selected. We analyzed their mathematics programs, considering every course that they require of their elementary teacher candidates. In all, we looked at 257 course syllabi and required textbooks as the source of information. Our sample represents elementary education schools at higher education institutions of all types and constitutes more than 5 percent of the institutions that offer undergraduate elementary teacher certification. Our analysis provides a reasonable assessment and the most comprehensive picture to date of how education schools are preparing — or failing to prepare — elementary teachers in mathematics.

In selecting this methodology, we understand that a course's intended goals and topics as reflected in syllabi and texts may differ from what actually happens in the classroom. We assert, however, that professors develop their syllabi and choose texts not for some empty purpose, but for quite an important one: to serve as an outline for the intended progression of a course and to articulate instructional objectives. We recognize that *less* than what the syllabi and certainly the texts contain, not more, is apt to be covered in class. The syllabus represents a professor's goal for what he or she wishes to accomplish in a course; in reality, however, there are the inevitable interruptions and distractions that almost always leave that goal to some degree unmet. We acknowledge the inherent limitations of this methodology and for this reason, *twice* invited the selected schools to submit additional materials, such as final exams and study guides, in order to enhance our understanding. Also, when we encountered any sort of ambiguity, we always gave the school the benefit of the doubt. Given the extremely low threshold that we set for schools to earn a good rating, we expected many more schools to pass than ultimately did.

How were schools rated? We considered three factors:

1. **RELEVANCE**: Does the education school require coursework that is relevant to the job of the elementary teacher, as opposed to coursework requirements intended for any student on the campus?
2. **BREADTH**: Does the coursework cover essential mathematics topics?
3. **DEPTH**: Is enough time available to devote sufficient attention to the essential topics?

Unfortunately, we could not evaluate schools on the basis of a fourth factor: rigor. Materials we obtained from schools did not allow us to do a comprehensive evaluation of whether they delivered a college-level program in elementary mathematics content, as opposed to offering a remedial program.

Though the full report contains an extensive discussion of all three criteria, some attention here is needed to explain the first: relevance. At the outset of this study, we presumed, as we know many people do, that while elementary teachers should be required to take some mathematics at the college level, it did not really matter what those courses were. The logic behind this approach is that if a teacher candidate can pass a college-level, general-audience mathematics course, then he or she should not have much difficulty wrestling with mathematics as an instructor in an elementary classroom. Any instructional strategies that a teacher needs to know could be taught in a mathematics methods course.

Nevertheless, every expert we consulted told us we were wrong. With remarkable consensus, mathematicians and mathematics educators believe that the "anything goes" practice of educating aspiring elementary teachers is both inefficient and ineffective. While perhaps counterintuitive, it is indeed university mathematicians who lead the charge against these general-audience mathematics courses, arguing instead that elementary teacher candidates need a rigorous program of study that returns them to the topics they encountered in elementary and middle school grades, but which is by no means remedial.

To better illustrate what the learning objectives would be for such courses, we created a tear-out test containing the kinds of mathematics problems that should be taught to teacher candidates and which they should be able to solve. The full test is available at www.nctq.org. A few sample problems follow.

SAMPLE PROBLEMS

EXIT WITH EXPERTISE:

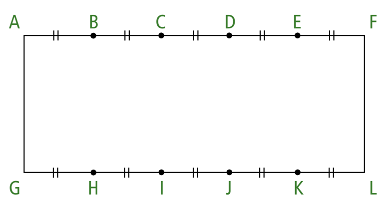
Do Ed Schools Prepare Elementary Teachers to Pass This Test?

(Answer Key can be found on page 21. The complete test is available at www.nctq.org.)

1. A store has a sale with a d% discount and must add a t% sales tax on any item purchased. Which would be cheaper for any purchase:
   a. Get the discount first and pay the tax on the reduced amount.
   b. Figure the tax on the full price and get the discount on that amount.
   Justify your answer.

2. Let n be an odd number.
   a. Prove that n^2 is odd.
   b. Prove that when n^2 is divided by 4, the remainder is 1.
   c. Prove that when n^2 is divided by 8, the remainder is 1.
   d. Find an odd n such that n^2 divided by 16 leaves a remainder that is not 1.

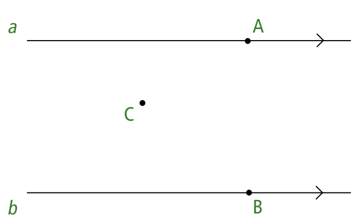
3. John's shop sells bicycles and tricycles. One day there are a total of 176 wheels and 152 pedals in the shop. How many bicycles are available for sale in John's shop that day? Solve arithmetically and algebraically.

4. 

   Let b represent the base of the rectangle and h represent its height.

   A different polygon is drawn within each of three rectangles with vertices AFLG.
   Polygon No. 1: A parallelogram with vertices DFIG
   Polygon No. 2: A trapezoid with vertices EFJG
   Polygon No. 3: A triangle with vertices ALH
   How do the areas of the three polygons compare? Justify your answer.

5. Lines a and b are parallel. Connect points A and C, and points B and C with line segments. The measurement of the acute angle with its vertex at point B created by $\overline{CB}$ is 40º; the measurement of the acute angle created by $\overline{CA}$ with its vertex at point A is 30º. Find the measurement of ∠ACB.



FINDINGS

FINDING 1:

Few education schools cover the mathematics content that elementary teachers need. In fact, the education schools in our sample are remarkable for having achieved little consensus about what teachers need. There is one unfortunate area of agreement: a widespread inattention to algebra.

The variation in requirements across the sample 77 education schools, all preparing individuals to do the same job, is unacceptable, and we suspect reflects the variation found across all American education schools.

Depending upon the institution, elementary teacher candidates are required to take anywhere from zero to six mathematics courses in their undergraduate careers. The content of this coursework ranges from "Integrated Mathematics Concepts" (described as a survey course in contemporary mathematics that presents mathematics as a human endeavor in a historical context) to "Calculus."

Within this variation, few education schools stand out for the quality of their mathematics preparation. Only ten schools in our sample (13 percent) rose to the top in our evaluation of the overall quality of preparation in mathematics. These schools met all three of our criteria: relevance, breadth, and depth.

The table on page 7 lists the institutions by rankings. With the exception of the University of Georgia, which we single out as an exemplary program, the listings are in alphabetical order within the group rankings.

ARE EDUCATION SCHOOLS PREPARING ELEMENTARY TEACHERS TO TEACH MATHEMATICS?

Education Schools with the Right Stuff

An exemplary teacher preparation program

UNIVERSITY OF GEORGIA

Boston College, MA†
Indiana University, Bloomington
Lourdes College, OH†
University of Louisiana at Monroe
University of Maryland, College Park
University of Michigan
University of Montana†
University of New Mexico†
Western Oregon University†

† Although these schools pass for providing the right content, they still fall short on mathematics methods coursework. They do not require a full course dedicated solely to elementary mathematics methods.

Education Schools that Would Pass if They Required More Coursework

Arizona State University
Boston University
Calumet College of St. Joseph, IN
Cedar Crest College, PA
Chaminade University of Honolulu, HI
Columbia College, MO
Concordia University, OR
Georgia College and State University
King's College, PA
Lewis-Clark State College, ID
Minnesota State University Moorhead
Radford University, VA
Saint Joseph's College of Maine
Saint Mary's College, IN
Southern New Hampshire University
State University of New York (SUNY) College at Oneonta
University of Central Arkansas
University of Louisville, KY
University of Mississippi
University of Nevada, Reno
University of Portland, OR
University of South Carolina
University of South Dakota
University of Texas at El Paso
University of Wyoming
West Texas A&M University

Education Schools that Would Pass with Better Focus and Textbooks

Benedictine University, IL
Northeastern State University, OK
The College of New Jersey
Towson University, MD
Western Connecticut State University
Wilmington University, DE

Education Schools that Fail on All Measures

Albion College, MI
American University, DC
California State University, San Marcos*
California State University, Stanislaus*
Colorado College*
Florida International University
Green Mountain College, VT**
Greensboro College, NC*
Gustavus Adolphus College, MN*
Hampton University, VA*
Iowa State University
Lee University, TN
MacMurray College, IL
Metropolitan State College of Denver, CO
Newman University, KS
Norfolk University, VA
Park University, MO
Seattle Pacific University, WA
Southern Adventist University, TN*
St. John's University, NY*
Saint Joseph's University, PA*
University of Alabama at Birmingham*
University of Arizona
University of Memphis, TN
University of Nebraska at Omaha
University of New Hampshire, Durham
University of Redlands, CA*
University of Rhode Island*
University of Richmond, VA*
University of Texas at Dallas
Utah State University
Valley City State University, ND
Viterbo University, WI
Walla Walla College, WA
West Virginia University at Parkersburg

* Programs requiring no elementary content coursework at all.

** New coursework requirements are not publicly available.

Improving the Heft and Focus of Mathematics Preparation for Elementary Teachers

A fundamental problem observed in most of the programs is that there is a large deficit in the amount of time devoted to elementary mathematic topics. We considered the time spent on the four critical areas of mathematics that an elementary teacher needs to understand: 1) numbers and operations, 2) algebra, 3) geometry and measurement, and 4) data analysis and probability. The table below shows how much programs deviate from the recommended time allocation. Of the four areas, algebra instruction is most anemic: over half of all schools (52 percent) devote less than 15 percent of class time to algebra, with another third effectively *ignoring it entirely*, devoting less than 5 percent of class time to that area. By a number of measures, including the recommendation of our Mathematics Advisory Group, algebra should comprise a large part of an entire elementary content course, roughly 25 percent of the preparation in mathematics that elementary teachers receive. While elementary teachers do not deal explicitly with algebra in their instruction, they need to understand algebra as the generalization of the arithmetic they address while studying numbers and operations, as well as algebra's connection to many of the patterns, properties, relationships, rules, and models that will occupy their elementary students. They should learn that a large variety of word problems can be solved with either arithmetic or algebra and should understand the relationship between the two approaches.

Deficiencies in Mathematics Instruction for Teachers

Critical areas	Recommended distribution (hours)	Average hours shortchanged (Estimated for the sample.)
Numbers and operations	40	13
Algebra	30	24
Geometry and measurement	35	14
Data analysis and probability	10	1

FINDING 2:

States contribute to the chaos. While most state education agencies issue guidelines for the mathematics preparation of elementary teachers, states do not appear to know what is needed.

Since all aspects of public K-12 education in the United States are regulated by the states, regulation of the preparation of K-12 teachers, whether at private or public colleges, is also within the purview of states. Even without national oversight states could be more consistent in their requirements regarding coursework, standards, and/or preparation for assessments in specific areas of mathematics.

States' Guidance is Confusing

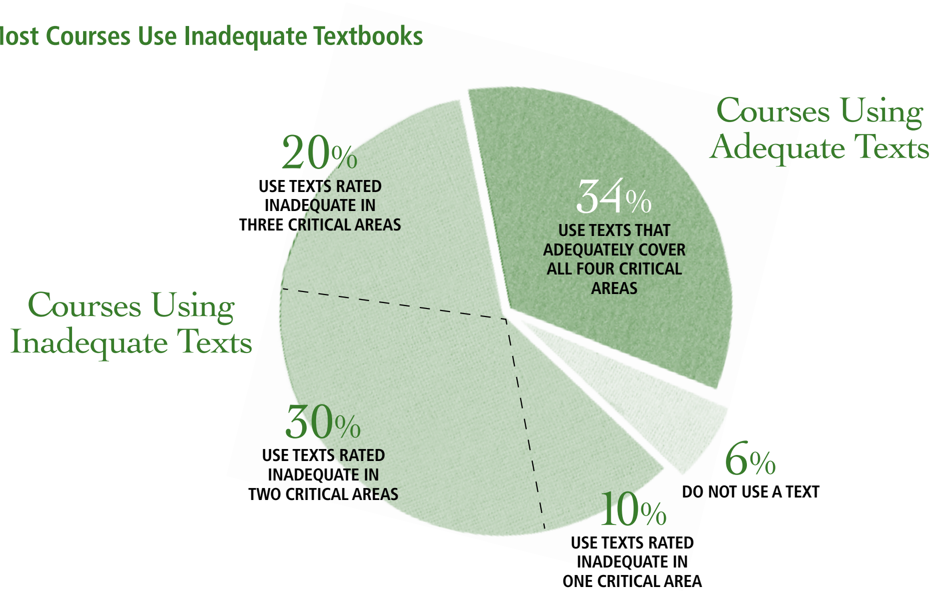
18	states have no requirements or no requirements pertaining to specific areas of math:	Alabama, Arizona, Arkansas, Connecticut, Hawaii, Idaho, Iowa, Louisiana, Maine, Maryland, Michigan, Mississippi, Missouri, Nebraska, New Jersey, Virginia, Wisconsin, and Wyoming
1	state has requirements pertaining only to geometry:	Minnesota
3	states have requirements pertaining only to foundations of mathematics and geometry:	Colorado, North Carolina, and Oregon
29	states have requirements pertaining to foundations of mathematics, algebra, and geometry:	Alaska, California, Delaware, District of Columbia, Florida, Georgia, Illinois, Indiana, Kansas, Kentucky, Massachusetts, Montana, Nevada, New Hampshire, New Mexico, New York, North Dakota, Ohio, Oklahoma, Pennsylvania, Rhode Island, South Carolina, South Dakota, Tennessee, Texas, Utah, Vermont, Washington, and West Virginia

Source: NCTQ's *State Teacher Policy Yearbook 2007*, www.nctq.org/stpy

FINDING 3:

Most education schools use mathematics textbooks that are inadequate. The mathematics textbooks in the sample varied enormously in quality. Unfortunately, two-thirds of the courses use no textbook or a textbook that is inadequate in one or more of four critical areas of mathematics. Again, algebra is shortchanged, with no textbook providing the strongest possible support.

Most Courses Use Inadequate Textbooks



Only one-third of the elementary content courses in our sample use a textbook that was rated as adequate in four critical areas of mathematics (numbers and operations; algebra; geometry and measurement; and data analysis and probability). Predictably, the algebra portions of the textbooks are the weakest, with the majority of textbooks earning scores low enough to label them unacceptable for use in algebra instruction.

In fact, no textbook has the strongest possible stand-alone algebra section, a fact that will handicap the preparation of elementary teachers in this vital area.

FINDING 4:

Almost anyone can get in. Compared to the admissions standards found in other countries, American education schools set exceedingly low expectations for the mathematics knowledge that aspiring teachers must demonstrate.

Sixteen percent of the education schools do not require applicants to pass any sort of mathematics test to get into their programs. The majority of the *77* schools require applicants to take a form of **basic skills test** for admission, typically a three-part assessment of skills in reading, writing, and mathematics. None of these tests, including the most popular choice, the Praxis I, measures the proficiency one should expect from a high school graduate, as they address only those mathematics topics taught in elementary and middle school grades.[1]

Entrance Tests on Mathematics Knowledge[2]

No. of schools	Do they have tests?
11	No test at all
14	Test requirements or test expectations not clear
54	Basic skills test
1[3]	Test for high school proficiency

Only one schoool in our sample of 77 clearly has adequate entry requirements.

1 We classify algebra as a middle school course because it is such in most developed countries.
2 The total number of schools noted in the table is more than *77* because some schools have multiple options for entrance tests.
3 Colorado College requires applicants to score at least 600 on the SAT math.

FINDING 5:

Almost anyone can get out. The standards used to determine successful completion of education schools' elementary teacher preparation programs are essentially no different than the low standards used to enter those programs.

Most education schools told us that they require an exit test in mathematics. In almost all cases, these exit tests are the same tests that teachers need to take for state licensure (Praxis II or a test specific to a state). There are two major failings of these tests: they either do not report a subscore for the mathematics portion of the test, or if they do report a mathematics subscore, it is not a factor in deciding who passes. Under these circumstances it may be possible to answer nearly every mathematics question incorrectly and still pass the test.

The fact that education schools are relying on tests that allow prospective teachers to pass without demonstrating proficiency in all subject areas with "stand-alone" tests makes it impossible for either the institution or the state in which they are going to teach to know how much mathematics elementary teachers know at the conclusion of their teacher preparation program.

In addition, even if these tests require a demonstration of mathematical understanding of slightly more depth than entrance tests, it is insufficient to establish whether elementary teacher candidates are truly prepared for the challenges of teaching mathematics.

Exit Tests on Mathematics Knowledge

No. of schools	Do they have tests?
17	No test at all or test requirement not clear
60[1]	Only assess elementary and middle school proficiency and do not use a stand-alone test
0[2]	Stand-alone test for what an elementary teacher needs to know

Not a single state requires an adequate exit test to ensure that the teacher candidate knows the mathematics he or she will need.

1 California's licensing test (CSET) appears to be the most rigorous of these tests, but the mathematics portion is not stand-alone.
2 Massachusetts plans to unveil in winter 2009 a stand-alone test of the mathematics an elementary teacher needs to know.

The Other Dimension of Mathematics Preparation: Mathematics Methods Coursework

Our study focused primarily on the content preparation of elementary teachers in mathematics, but courses in which aspiring teachers learn the methods of mathematics instruction are essential in their overall preparation for the classroom. Therefore we also examined mathematics methods coursework, in particular whether it was generally adequate and how instructors designed practice teaching experiences to ensure that teacher candidates focused on conveying mathematics to their child audiences.

FINDING 6:

The *elementary mathematics* in mathematics methods coursework is too often relegated to the sidelines. In particular, any practice teaching that may occur fails to emphasize the need to capably convey mathematics content to children.

Many mathematics educators report that it is difficult to adequately cover all elementary topics in even one methods course, yet a large share of the education schools we studied (42 percent) do not have even one methods course dedicated to elementary mathematics methods and 5 percent have only a two credit course. Looking at programs that had a course devoted solely to elementary mathematics methods and required practice teaching, we found only six education schools that appeared to emphasize the need for aspiring teachers to consider how to communicate mathematical content and how to determine if children understood what they had been taught:

Education schools which put *mathematics* at the center of practice teaching

1. Greensboro College
2. University of Georgia
3. University of Louisville
4. University of Michigan
5. University of Nevada, Reno
6. University of Texas at El Paso

In the methods syllabi found in these six programs we saw instructor expectations for practice teaching such as this: *The student has demonstrated an appreciation of what it means to teach mathematics for conceptual understanding*. In contrast, syllabi from other courses requiring practice teaching tended to make the mathematics instruction almost beside the point. For example, an aspiring teacher might be asked to answer a question such as: *What part of your teaching philosophy did you demonstrate in your experience?*

FINDING 7:
Too often, the person assigned to teach mathematics to elementary teacher candidates is not professionally equipped to do so. Commendably, most elementary content courses are taught within mathematics departments, although the issue of just who is best qualified and motivated to impart the content of elementary mathematics to teachers remains a conundrum.

No matter which department prepares teachers in mathematics, elementary content mathematics courses must be taught with integrity and rigor, and not perceived as the assignment of the instructor who drew the short straw. The fact that prospective teachers may have weaker foundations in mathematics and are perceived to be more math phobic than average should not lead to a conclusion that the mathematics presented must be watered down.

FINDING 8:
Almost anyone can do the work. Elementary mathematics courses are neither demanding in their content nor their expectations of students.

We could not evaluate the rigor in mathematics content courses taught in our sample education schools using syllabi review because too few syllabi specified student assignments. We did, however, make use of assessments that some education schools provided us. With a cautionary note that these assessments may not be representative of all the schools in our sample, their general level of rigor is dismaying.

The table on page 15 demonstrates the contrast between two types of questions taken from actual quizzes, tests, and exams used in courses in programs in our sample. It pairs three problems that would be appropriate for an elementary classroom with three problems appropriate for a college classroom, both on a related topic.

About a third of the questions in assessments we obtained from mainstream education schools were completely inappropriate for a college-level test.

CONTRASTING PROBLEMS: The mathematics that teachers need to know – and children do not

Mathematics questions CHILDREN should be able to answer – taken from actual college course assessments.

1a. The number 0.0013 is equal to the following:

a. thirteen thousandths
b. thirteen ten-thousandths
c. zero point one three
d. one hundredth and three ten-thousandths

2a. Which of the following is (2 ½) ÷ (1/2)?

a. 1 ¼ b. 2 ¼ c. 1 ½ d. 5

3a. Exactly three-fourths of the students in a certain class are passing. If 24 of them are passing, how many students are in the course?

a. 18 b. 32 c. 36 d. 42

Mathematics questions that are closer to hitting the mark for what TEACHERS should be able to answer – taken from actual college course assessments.

1b. Solve the problem and explain your solution process. Write the number 1.00561616161… as a quotient of two integers (that is, in fractional-rational form). Show step-by-step arithmetic leading to your final, answer, giving a teacher-style solution. Do not simplify your final answer.

2b. Simplify the fraction

$$\frac{(1/2 + 1/3) \div (5/12)}{(1 - 1/2)\,(1 - 1/3)\,(1 - 1/4)}$$

3b. The big dog weighs 5 times as much as the little dog. The little dog weighs 2/3 as much as the medium sized dog. The medium dog weighs 9 pounds more than the little dog. How much does the big dog weigh? Solve the problem and explain your solution process.

RECOMMENDATIONS

We suspect that in several decades we will look back on the current landscape of the mathematics preparation of elementary teachers and have the benefit of hindsight to realize that some education schools were poised for significant and salutary change. These are the schools that now have the basic "3/1" framework already in place for adequate preparation, that is, **three** mathematics courses that teach the elementary mathematics content that a teacher needs to know and **one** well-aligned mathematics methods course. Our recommendations here are addressed to professionals responsible for elementary teacher preparation: professional organizations, states, education schools, higher education institutions, and textbook publishers. We also propose initiatives that would build on the 3/1 framework in order to achieve a truly rigorous integration of content and methods instruction.

THE ASSOCIATION OF MATHEMATICS TEACHER EDUCATORS (AMTE)

The Association of Mathematics Teacher Educators (AMTE) should organize mathematicians and mathematics educators in a professional initiative and charge them with development of prototype assessments that can be used for course completion, course exemption, program completion, and licensure. These assessments need to evaluate whether the elementary teacher's understanding of concepts such as place value or number theory is deep enough for the mathematical demands of the classroom. They should be clearly differentiated from those assessments one might find in an elementary or middle school classroom.

We offer a sample test, *Exit with Expertise: Do Ed Schools Prepare Elementary Teachers to Pass This Test?* (an excerpt is on page 5 and the full test is available on our website: www.nctq.org) as a jumping-off point for the development of a new generation of tests that will drive more rigorous instruction and ensure that teachers entering the elementary classroom are well prepared mathematically.

STATES

States must set thresholds for acceptable scores for admission to education schools on standardized achievement tests, college placement tests, and high school exit tests. The guiding principle in setting these scores should be to ensure that every teacher candidate possesses a competent grasp of high school geometry and second-year high school algebra.

While these proposed thresholds are significantly higher than current ones, they are reasonable. In fact, they still may be lower than what is required of elementary teachers in nations reporting higher levels of student achievement in mathematics. With the exception of the most selective institutions, there is a quite plausible perception that an education school cannot raise its admission standards without putting itself at a disadvantage in the competition for students. The pressure these institutions face to accept a sufficient number of students makes it incumbent upon states to raise the bar for all education schools, not just relegate the task to a few courageous volunteers.

States need to develop strong coursework standards in all four critical areas: numbers and operations, algebra, geometry and measurement, and data analysis and probability.

States need to adopt wholly new assessments, not currently available from any testing company, to test for these standards.

A unique stand-alone test of elementary mathematics content that a teacher needs to know is the only practical way to ensure that a state's expectations are met. The test could also be used as a vehicle to allow teacher candidates to test out of required coursework.

States need to eliminate their PreK-8 certifications. These certifications encourage education schools to attempt to broadly prepare teachers, in the process requiring too few courses specific to teaching any grade span.

Currently, 23 states offer some form of PreK-8 certification.

EDUCATION SCHOOLS

Education schools should require coursework that builds towards a deep conceptual knowledge of the mathematics that elementary teachers will one day need to convey to children, moving well beyond mere procedural understanding. For most programs, we recommend a 3/1 framework: three mathematics courses designed for teachers addressing elementary and middle school topics and one mathematics methods course focused on elementary topics and numbers and operations in particular.

Teacher preparation programs should make it possible for an aspiring teacher to test out of mathematics content course requirements using a new generation of standardized tests that evaluate mathematical understanding at the requisite depth.

The higher education institutions in our sample require an average of 2.5 courses in mathematics, only slightly below our recommendation of three elementary content mathematics courses, although much of that coursework bears little relation to the mathematics that elementary teachers need. Institutions, provided they are willing to redirect their general education requirements to more relevant coursework for the elementary teacher, can quickly move towards meeting this standard by substituting requirements for elementary content mathematics courses.

Algebra must be given higher priority in elementary content instruction.

As the National Mathematics Advisory Panel made clear in its 2008 report, while proficiency with whole numbers, fractions, and particular aspects of geometry and measurement are the "critical foundation of algebra," adequate preparation of elementary students for algebra requires that their teachers have a strong mathematics background in those critical foundations, as well as algebra topics typically covered in an introductory algebra course.

Education schools should eliminate any of the following: mathematics programs designed for too many grades, such as PreK-8; the practice of teaching methods for science or other subjects as companion topics in mathematics methods coursework; and the practice of combining content and methods instruction if only one or two combined courses are required.

Teacher preparation programs do a disservice to the material that future elementary teachers need to learn by trying to accomplish too many instructional goals at the same time.

Five-year programs, such as those found in California, need to be restructured if they are going to meet the mathematics content needs of elementary teachers.

The five-year model for teacher preparation, whereby prospective teachers complete coursework for an undergraduate major taking the same courses as would any other major in that subject and than devote a fifth year to courses about teaching and learning, does not accommodate coursework designed for teachers in elementary mathematics topics. For that reason, these programs as currently structured are inadvisable for the appropriate preparation of elementary teachers for teaching mathematics.

HIGHER EDUCATION INSTITUTIONS

On too many campuses, teacher preparation is regarded by university professors and administrators as a program that is beneath them and best ignored. The connection of our national security to the quality of the teachers educating new generations of Americans goes unrecognized. Were education schools to receive more university scrutiny, and demands made that they be more systematic — neither of which is an expensive proposition — change could be dramatic.

Higher education institutions housing education schools must take the lead in orchestrating the communication, coordination, and innovation that would make the mathematics preparation of elementary teachers coherent.

Much of what has to be changed about the preparation of teachers connects to decisions regarding instruction in mathematics courses (e.g., textbook selection, the priority attached to algebra, establishing more rigorous standards) and mathematics methods courses (e.g., coordination with content courses, possibly through concurrent registration, emphasizing the *mathematics* in mathematics methods, especially in practice teaching). Many changes cannot be made in isolation and most will not be undertaken without explicit encouragement by institutional leadership.

TEXTBOOK PUBLISHERS

Several elementary content textbooks (particularly those by Thomas Parker and Scott Baldridge, and Sybilla Beckmann) are excellent and we recommend their use, but content textbooks that are more consistently good across all topics are still needed.

Professionals dedicated to improvements in elementary teacher preparation should collaborate to develop a textbook that can serve as a resource both in content and methods coursework. This ideal "combo-text" would augment a core of solid mathematics content with discussion of a process for continuous improvement of instruction focused on student learning.

CONCLUSION

American elementary teachers as a group are caring people who want to do what is best for children. Unfortunately, their mathematics preparation leaves far too many of them ill-equipped to do so. We are confident that the education schools that rose to the top in our evaluation process are preparing teachers *relatively* well compared to the majority of education schools in this study which rated so poorly. Their teachers stand readier than most to forestall the frustrations of youngsters leaving the familiar world of the counting numbers or dealing with the debut of division with fractions. Nonetheless, the standards against which these education schools were judged only lay a solid foundation. Further improvement is still necessary.[1] Until such time as an improved instructional model is developed that combines mathematics content and mathematics methods instruction, teacher preparation programs should increase the efficacy of existing content courses:

- Intensifying teacher preparation on essential topics with the same "laserlike focus" endorsed by the National Mathematics Advisory Panel for K-12 mathematics instruction.
- Selecting the best of current textbooks.
- Setting high standards for student performance in courses and in exit tests.

A deeper understanding of elementary mathematics, with more attention given to the foundations of algebra, must be the new "common denominator" of our preparation programs for elementary teachers within education schools. But we are only at the beginning of the process of seeing how that new measure might be calculated.

1 The prospect that mathematics specialists will become increasingly common in elementary classrooms due to initiatives promoted by groups including the National Academies (*Rising Above the Gathering Storm: Energizing and Employing America for a Brighter Economic Future*, Washington D.C., National Academic Press, 2007) does not change this imperative for improvement since those specialists can emerge from the same courses and programs as regular elementary classroom teachers. The reforms that will make classroom teachers more mathematically competent could improve mathematics specialists as well.

ANSWER KEY FOR SAMPLE PROBLEMS ON PAGE 5

EXIT WITH EXPERTISE:
Do Ed Schools Prepare Elementary Teachers to Pass This Test?
(The complete test is available at www.nctq.org.)

1. Neither is cheaper since both approaches yield the same total purchase price. To determine this, let p represent any purchase price:

   a. Discounted price: $p - p*(d/100) = p(1-d/100)$
   Tax on discounted price: $p(1-d/100)\ (t/100)$
   Adding the two and simplifying: $p(1-d/100) + p(1-d/100)(t/100) = p(1-d/100)(1 + t/100)$

   b. Full price with tax: $p+p*(t/100) = p(1+t/100)$
   Discount on full price with tax: $[p+p*(t/100)]*d/100 = p(1+t/100)(d/100)$
   Subtracting the discount from the full price and simplifying: $p(1+t/100) - p(1+t/100)(d/100) = p(1+t/100)(1-d/100)$

   These are the same since $a*b = b*a$

2. If n is an odd number, it can be represented as $2w+1$, where w represents a whole number (0,1,2…).

   a. $n^2 = (2w+1)^2 = 4w^2+4w+1 = 2(2w^2+2w) + 1$ so n^2 is odd.

   Helpful reminder for (b) and (c): In division with a remainder, when dividing by a number k, the result is a whole number and a remainder, with the remainder less than k (and greater than or equal to 0).

   b. $n^2 = (2w+1)^2 = 4w^2+4w+1 = 4(w^2+w) + 1$
   Since w^2+w is a whole number and 1 is less than 4, the remainder when dividing by 4 is 1.

   c. $n^2 = (2w+1)^2 = 4w^2+4w+1 = 8[(w^2+w)/2] + 1$. The expression $w^2+w = w(w+1)$, and either w or $w+1$ is even, so $(w^2+w)/2$ is a whole number. Thus the remainder when dividing by 8 is 1.

   d. Many odd numbers when their square is divided by 16 leave a remainder that is not 1. The number 3 is the least odd number that satisfies this condition: $3^2 = 9$, and when this is divided by 16 the remainder is 9.

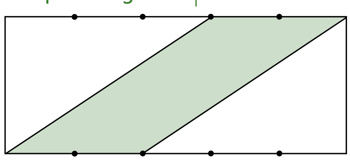
3. There are 52 bicycles in the shop.

   Solved arithmetically:
   Each bicycle has two wheels and each tricycle has three wheels, and both have two pedals. For each tricycle, there is one more wheel than pedals. There are 176−152=24 extra wheels, so there are 24 tricycles. These have 24*3=72 wheels, so the number of wheels on bicycles is 176−72=104. The number of bicycles is half the number of wheels, 104/2=52.

   Solved algebraically:
   Let b represent the number of bicycles in the store and t the number of tricycles.
   Equation A, developed using number of wheels: $2b+3t = 176$
   Equation B, developed using number of pedals: $2b+2t = 152$
   Subtracting equation B from A: $1t = 24$
   Substituting this value for t into equation B and solving for b, $b = 52$

4. All the polygons have the same area: $A_1 = A_2 = A_3$

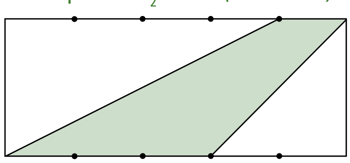
   Area of parallelogram: $A_1 = \frac{2}{5}b*h$



h

b

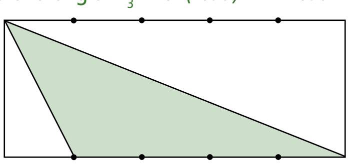
   Area of trapezoid: $A_2 = \frac{1}{2}h\,(\frac{3}{5}b + \frac{1}{5}b) = \frac{1}{2}h * \frac{4}{5}b = \frac{2}{5}b*h$



h

b

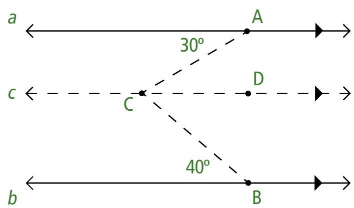
   Area of triangle: $A_3 = \frac{1}{2}(\frac{4}{5}b)*h = \frac{2}{5}b*h$



h

b

5.



Angle ACB measures 70º.

Different approaches are possible, but one approach is to draw an auxiliary line[1] parallel to lines *a* and *b* through point C and add point D to line *c*:

$m\angle ACD = 30°$ (This is an alternate interior angle to the acute angle with vertex A on line *a*.)

$m\angle DCB = 40°$ (This is an alternate interior angle to the acute angle with vertex B on line *b*.)

$m\angle ACD + m\angle DCB = m\angle ACB = 30° + 40° = 70°$

1 The function of auxiliary lines is to change difficult probelms to simpler ones, often ones which have already been solved. Auxiliary lines could also be drawn perpendicular to line *a* through point A, creating a quadrilateral whose angles include ∠ACB and can be solved, or perpendicular to line *c* through point C, creating two triangles, the solution of whose angles resolves the measurement of ∠ACB. An auxiliary line can also be drawn through points B and C; its intersection with line *a* creates a triangle, the solution of whose angles resolves the measurement of ∠ACB.

"I commend this valuable report from the National Council on Teacher Quality for addressing a critical need in improving teacher capacity: more effective assessments of mathematical knowledge as part of the process by which candidates qualify for entry into elementary teacher preparatory programs."

— Larry R. Faulkner
President, Houston Endowment Inc.
President Emeritus of the University of Texas

"This report should help counter the common belief that the only skill needed to teach second-grade arithmetic is a good grasp of third-grade arithmetic. Our education schools urgently need to ensure that our elementary teachers do not represent in the classroom the substantial portion of our citizenry that is mathematically disabled. We must not have the mathematically blind leading the blind."

— Donald N. Langenberg
Chancellor Emeritus, University of Maryland

To download the full report, go to www.nctq.org.
For additional copies of the executive summary, contact:

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